Context where hyperfunction is introduced

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(1) Context known by many people

When road or railway passes a river, a bridge is built. As many people use road or railway on a daily basis, they look at and know bridges. If we start to discuss by ob-serving a bridge, many people may understand the context. When we simplify a bridge, we suppose a log bridge. This report discusses load applied on a log bridge, and introduces hyperfunction.

Shown is a figure of a log bridge.

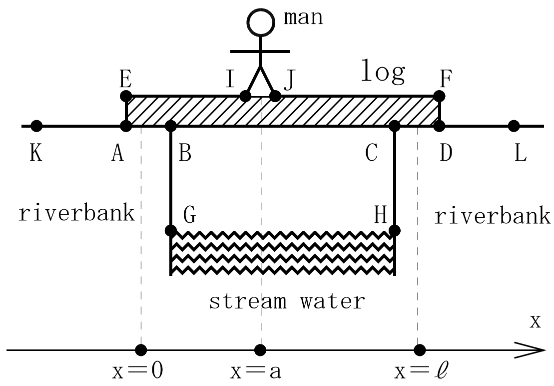

Fig. log bridge

Point B and point C are both banks of a river, line KA and line DL are ground surface contiguous with the log bridge. Line GH is the stream water surface of the river. As length AB and length CD are very short, they are recognized as a point respectively. In order to express locations, coordinate x is set, and the location of point AB be $x=0$, the location of point CD be $x=\ell$. The unit of coordinate x is m. The length of the log bridge is ℓm. Length AE expresses diameter of the log, and suppose the diameter be constant. As the diameter AE of the log is very short compared to the length ℓ of the log, the diameter be neglected and the log is recognized as a line. A man is walking on the log bridge, and the line range IJ is the man, As length IJ is very short, it is recognized as a point. The location of point IJ be $x=a$.

(2) Loads of different units

Weight of a man applies a force to downward direction on a log bridge. The unit of force is N. Force applied on a bridge is called load. Load of a man is recognized as a force of magnitude P·N which applies at a point $x=a$. Weight of the log becomes a force to downward direction, and applies on a log bridge. As the log is the bridge itself, the load of the log is called own weight. As load of the log applies on the range from point $x=0$ to point $x=\ell$ uniformly, it is recognized as a force per unit length of magnitude W·N/m. Load of a man is recognized the unit of N, load of a log is recognized the unit of N/m, the units are different.

(3) Concentrated load and distrbuted load

When load of a man applying at point $x=a$ is expressed by $g(x)$, it is expressed as formula (1), formula (2).

$$g(x)=P \qquad (x=a) \tag{1}$$

$$g(x)=0 \qquad (0\leqq x<a、a<x\leqq \ell) \tag{2}$$

As load of a man applies only at point $x=a$, and doesn't apply any other point, it is called concentrated load. Concentrated load is expressed by discrete function like formula (1), formula (2). When load of a log applying at point x is expressed by function $h(x)$, it is expressed as formula (3).

$$h(x)=W \qquad (0\leqq x\leqq \ell) \tag{3}$$

As load of a log is dispersed on the whole interval $0\leqq x\leqq \ell$, it is called distributed load. Ditrbuted load is expressed by continuous function like formula (3).

(4) Not simple distrbution

Distrbution is explained specifying field of distribution and distrbuted quantity. Log bridge is recognized as a line neglecting its thickness and is unidimentional space. Load applying a log bridge is a distrbution, and field of distribution is unidimentional space expressed by coordinate x, and distributed quantity is load of a man g(x) and own weight of a log bridge h(x). Ditributed load is composed of 2 with different units, and it is not simple distribution. Each of load of a man g(x) and own weight of a log bridge h(x) is simple distribution with one unit, and is expressed by a function.

(5) Intrduction of component expression

Although load of a man g(x) and own weight of a bridge h(x) have different units, they impact similar effect on the log bridge and have close relevance. As they have close relevance, the auther tried to express them unified. Although unified load f(x) is superpsotion of load of a man g(x) and own weight of a log bridge h(x), it is not simple summation. It is expressed like formula (4) appending units.

$$f(x)\cdot N/m = h(x)\cdot N/m + g(x)\cdot N \qquad (4)$$

As discrete function g(x) is not object of differntial or integral calculation, assuming comtinuous function h(x) is representative, unit N/m is appended to unified load f(x). When formula (4) is divided unit N/m, formula (5) is obtained.

$$f(x) = h(x) + g(x)\cdot m \qquad (5)$$

Right side hand of formula (5) is thought to be a vector expression with basis vector of symbol 1 and symbol m, and symbol m has lost the meaning of length unit. When formula (5) is translated into numerical vector expression, formula (6) is obtained.

$$f(x) = \{h(x), g(x)\} \qquad (6)$$

In the expression of right side hand of formula (5), formula (6), h(x) and g(x) are called components.

(6) Differentiable approximate function

Although component g(x) is not object of differential or integral calculation, function G(x) of formula (7) which contains parameter ε is object of differential or integral calculation.

$$G(x) = \frac{P}{\varepsilon\sqrt{\pi}} \exp\{-(\frac{x-a}{\varepsilon})^2\} \qquad (7)$$

$\varepsilon > 0$ holds. Function g(x) of formula (1), formula (2) is obtained from function G(x) of formula (7) by calculation of formula (8) using parameter ρ.

$$g(x) = \lim_{\rho\to 0}\{\lim_{\varepsilon\to 0}\int_{x-\rho}^{x+\rho} G(t)\,dt\} \qquad (8)$$

$\rho > 0$ holds. Parameter ρ varries slower than parameter ε to limit. Function G(x) is called approximate function of function g(x). If function g(x) becomes object of differential or integral calculation through approcximate function G(x), symbol f(x) becomes object of differential or integral calculation, and symbol f(x) is called hyperfunction. Other than formula (7), there are a lot of functions G(x) each of them satisfies formula (8). Approximate functions each of them satisfies formula (8) are called equivalent to each other.

(7) Similar context

This report explains showing example of bridge, but there may be many other similar cases. There may be not simple distribution. Context may exist where items are closely related although units are different and they are reasonable to be dealt unified. Context may be reasonable where discrete function and continuous function are dealt unified. In order to discuss about these context, hyperfunction may be used as mathmatical means.