Sum, multiple, sideslip, inversion and differential of hyperfunction

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(1) Operation at each point

Sum, multiple, sideslip, inversion and differential are operations at each point. When function J(x) which is the result of operation performed on approximate functions is used as approximate function, components $j_h(x)$, $j_d(x)$, $j_1(x)$, $j_2(x)$, ・・・ are computed by formula(1)～formula(4)

$$j_h(x)=\lim_{\rho\to 0}\lim_{\varepsilon\to 0} J(x-\rho) \qquad (1)$$

$$j_d(x)=\lim_{\rho\to 0}\lim_{\varepsilon\to 0} \{J(x+\rho)-J(x-\rho)\} \qquad (2)$$

$$j_1(x)=\lim_{\rho\to 0}\lim_{\varepsilon\to 0} \int_{x-\rho}^{x+\rho} J(t)\,dt \qquad (3)$$

$$j_n(x)=\lim_{\rho\to 0}\lim_{\varepsilon\to 0} \int_{x-\rho}^{x+\rho} (t-x)^{n-1} J(t)\,dt \qquad (4)$$

Hyperfunction which is the result of operation is defined as formula(5) in which components $j_h(x)$, $j_d(x)$, $j_1(x)$, $j_2(x)$, ・・・ are seperated with cmma ″，″ lined up in brakets ″{ }″.

$$j(x)=\{j_h(x)、j_d(x)、j_1(x)、j_2(x)、・・・\} \qquad (5)$$

Multiple, sideslip, inversion and differential are operations performed on 1 hyperfunction f(x). Sum is an operation performed on 2 hyperfunctions f(x) and g(x). As far as sum, multiple, sideslip, inversion and differential are concerned, operations can be defined, because all components converge.

(2) Sum, multiple, sideslip

Function J(x) calculated by formula(6) defines sum of hyperfunctions f(x) and g(x). Function F(x) and G(x) are the approximate function of hyperfunction f(x) and g(x).

$$J(x)=F(x)+G(x) \qquad (6)$$

With respect to components, formula(7)～formula(10) holds.

$$j_h(x)=f_h(x)+g_h(x) \qquad (7)$$

$$j_d(x)=f_d(x)+g_d(x) \qquad (8)$$

$$j_1(x)=f_1(x)+g_1(x) \qquad (9)$$

$$j_n(x)=f_n(x)+g_n(x) \qquad (10)$$

Function J(x) calculated by formula(11) using constant c defines muitiple of hyperfunction of f(x).

$$J(x)=c\cdot F(x) \qquad (11)$$

With respect to components, formula(12)～formula(15) holds.

$$j_h(x)=c\cdot f_h(x) \qquad (12)$$

$$j_d(x)=c\cdot f_d(x) \qquad (13)$$

$$j_1(x)=c\cdot f_1(x) \qquad (14)$$

$$j_n(x)=c\cdot f_n(x) \qquad (15)$$

Function J(x) calculated by formula(16) using constant a defines sideslip of hyperfunction f(x).

$$J(x)=F(x-a) \qquad (16)$$

With respect to components, formula(17)～formula(20) holds.

$$j_h(x)=f_h(x-a) \qquad (17)$$

$$j_d(x)=f_d(x-a) \qquad (18)$$

$$j_1(x)=f_1(x-a) \qquad (19)$$

$$j_n(x)=f_n(x-a) \qquad (20)$$

As far as sum, multiple and sideslip are concerned, computing formulas of all components are the same form as computing formulas of approximate function.

Function J(x) calculated by formula(21) defines differnce of hyperfunctions f(x) and g(x).

$$J(x)=F(x)-G(x)=F(x)+(-1)\cdot G(x) \quad (21)$$

formula(21) is explained as combination of multiple of -1 and sum.

(3) Inversion

Function J(x) calculated by formula(22) defines inversion of hyperfunction f(x).

$$J(x)=F(-x) \quad (22)$$

With respect to components, formula(23)～formula(26) holds.

$$j_h(x)=f_d(-x)+f_h(-x) \quad (23)$$

$$j_d(x)=-f_d(-x) \quad (24)$$

$$j_n(x)=f_n(-x) \quad (n=1、3、5、\cdot\cdot\cdot) \quad (25)$$

$$j_n(x)=-f_n(-x) \quad (n=2、4、6、\cdot\cdot\cdot) \quad (26)$$

Among formula(23)～formula(26) which calculate components, only formula(25) is the same form as formula(22) which is computing formula of approximate function.

(4) Differentioal

Function J(x) calculated by formula(27) defines differntial of hyperfunction f(x).

$$J(x)=F'(x) \quad (27)$$

With respect to components, formula(28)～formula(32) holds.

$$j_h(x)=\lim_{\rho\to 0}(f_h)'(x-\rho) \quad (28)$$

$$j_d(x)=\lim_{\rho\to 0}\{(f_h)'(x+\rho)-(f_h)'(x-\rho)\} \quad (29)$$

$$j_1(x)=f_d(x) \quad (30)$$

$$j_2(x)=-f_1(x) \quad (31)$$

$$j_n(x)=-(n-1)f_{n-1}(x) \quad (32)$$

As far as differntial is conerned, among formula(28)～formula(32) which calculate components, no formula is the same form as formula(27) which is computing formula of approximate function. Left continuous component $j_h(x)$、step component $j_d(x)$ are expressed using $(f_h)'(x)$ which is derivative of left continuous component $f_h(x)$. First component $j_1(x)$ is coincides with step component $f_d(x)$. As for higher components than second, n-th component $j_n(x)$ is coincides with -(n-1) multiple of (n-1)-th component $f_{n-1}(x)$.

(5) Examples of calculation

From function J(x) expressed by formula (6), component $j_1(x)$ is computed as formula (33), substituting function J(x) into formula(3), and component $j_1(x)$ converges.

$$\begin{aligned} j_1(x) &= \lim_{\rho\to 0}\lim_{\varepsilon\to 0}\int_{x-\rho}^{x+\rho} J(t)\,dt \\ &= \lim_{\rho\to 0}\lim_{\varepsilon\to 0}\int_{x-\rho}^{x+\rho}\{F(t)+G(t)\}dt \\ &= \lim_{\rho\to 0}\lim_{\varepsilon\to 0}\int_{x-\rho}^{x+\rho}F(t)dt+\lim_{\rho\to 0}\lim_{\varepsilon\to 0}\int_{x-\rho}^{x+\rho}G(t)dt \\ &= f_1(x)+g_1(x) \end{aligned} \quad (33)$$

Formula(33) explains that formula(9) holds.

From function J(x) expressed by formula (22), component $j_h(x)$ is computed as formula(34), substituting function J(x) into formula(1), and component $j_1(x)$ converges.

$$\begin{aligned} j_h(x) &= \lim_{\rho\to 0}\lim_{\varepsilon\to 0}J(x-\rho)=\lim_{\rho\to 0}\lim_{\varepsilon\to 0}F(-x-\rho) \\ &= \lim_{\rho\to 0}\lim_{\varepsilon\to 0}F\{-(x+\rho)\} \\ &= \lim_{\rho\to 0}\lim_{\varepsilon\to 0}[F\{-(x+\rho)\}-F\{-(x-\rho)\}+F\{-(x-\rho)\}] \\ &= \lim_{\rho\to 0}\lim_{\varepsilon\to 0}[F\{-(x+\rho)\}-F\{-(x-\rho)\}] \\ &\quad +\lim_{\rho\to 0}\lim_{\varepsilon\to 0}F\{-(x-\rho)\} \\ &= f_d(-x)+f_h(-x) \end{aligned} \quad (34)$$

Formula(34) explains that formula(23) holds.

With respect to sum, multiple, sideslip, inversion and differential, components can be calculated by the similar way as formula(33)、formula(34).