Component expression conceived from observing mass distribution

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(1) Description of mass distribution

Suppose about mass through observing a pencil and a stone. Drill a pencil a small hole at position of lead and thread string through the hole. Drill a stone a small hole and thread string through the hole. Stretch the string and make it straight line. Then shown as Fig. Set

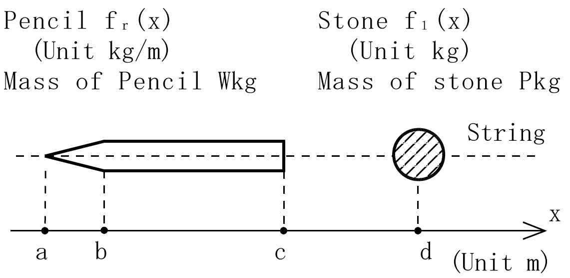


Fig. 2 kinds of distribution of mass

coordinate x along the string. As coordinate x is length, unit m is used.

When mass of pencil is considered, thickness is eliminated, but length is not able to eliminated. It is recognized that along a line which has length from point $x=a$ to point $x=c$, mass density of magnitude $f_r(x)$ is distributed. The unit of mass density $f_r(x)$ is kg/m. The part of pencil from point $x=a$ to point $x=b$ is sharpened.

When mass of stone is considered, size of stone is eliminated, and stone is considered as a point. It is recognized that at point $x=d$ where the stone is, mass of magnitude $f_1(x)$ is distributed. The unit of mass $f_1(x)$ is kg.

(2) Expression by continuous function

When mass density of pencil is expressed by function $f_r(x)$, function value is $f_r(a)=0$ at point $x=a$, function value increases gradually in interval $a \leqq x \leqq b$, function value becomes $f_r(b)$, at point $x=b$, function value is constant in interval $b \leqq x < c$, as expressed by formula (1).

$$f_r(x)=f_r(b) \qquad (b \leqq x < c) \qquad (1)$$

Except at point $x=c$, mass distribution of the pencil is expressed by continuous function. Integral W of formula(2) is the whole mass of the pencil.

$$W=\int_a^c f_r(x)\,dx \qquad (2)$$

As there is no pencil in interval $x \leqq a$, $c < x$, distribution of mass density is expressed by formula(3).

$$f_r(x)=0 \qquad (x \leqq a、c < x) \qquad (3)$$

(3) Unambiguity of step expression

At point $x=c$, cross section of pencil is perpendicular to the string. At point $x=c$, function value decreases suddenly from $f_r(b)$ of formula(1) to 0 of formula(3), therefore the value $f_r(c)$ is difficult to be identified as any of $f_r(b)$, 0, $\frac{1}{2}f_r(b)$, $\frac{1}{3}f_r(b)$, ・・・. Definition of function is that when one value of independent variable is decided, the value of dependent variable is settled responding to independent variable, This peculiarity is called unambiguity of function. Because the value $f_r(c)$ cannot be settled, density distribution $f_r(x)$ does not have unanbiguity at point $x=c$, as far as $f_r(x)$ is ordinary function. The point $x=c$ is called step point.

In order to conserve unambiguity of function, components are introduced about step point expression. Just left side of point $x=c$ is abbreviated to $x=c-0$, just right side is abbreviated to $x=c+0$. Function value $f_r(c-0)$ and $f_r(c+0)$ are exist, and they conserve unambiguity respectively. Left continuous component $f_h(c)$ is defined as formula(4) and step component $f_d(c)$ is defined as formula(5).

$$f_h(c)=f_r(c-0) \quad (4)$$

$$f_d(c)=f_r(c+0)-f_r(c-0) \quad (5)$$

Separating with comma , lining up putting in brackets ( ) mass density $f_0(x)$ is expressed as formula(6).

$$f_0(c)=\{f_h(c),f_d(c)\} \quad (6)$$

As formula(6) has 2 dependent variables $f_h(c)$ and $f_d(c)$, it is not ordinary function, but is the same expression as numerical vector. Thus formula(6) conserves unambiguity. Unit kg/m of component $f_h(c)$ and unit kg/m of component $f_d(c)$ are the same. At point $x=c$, function value $f_r(c)$ is difficult to settle, but function value $f_0(c)$ is written as formula(7) with 2 components.

$$f_0(c)=\{f_r(b),-f_r(b))\} \quad (7)$$

(4) Unification of continuous and step

As for interval $x\neq c$, mass density is modified from function $f_r(x)$ to component expression $f_0(x)$, define mass density $f_0(x)$ using formula(8), formula(9), formula(10) which are the same type as formula(4), formula(5), formula(6), the expression of continuous and step is unified.

$$f_h(x)=f_r(x-0) \quad (8)$$

$$f_d(x)=f_r(x+0)-f_r(x-0) \quad (9)$$

$$f_0(x)=\{f_h(x),f_d(x)\} \quad (10)$$

As for interval $x\neq c$, function value $f_r(x)$ exist in addition to $f_r(x-0)$ and $f_r(x+0)$, and they conserve unambiguity. As for interval $x\neq c$, when formula(8) is calculated formula(11) holds, when formula (9) is calculated formula(12) holds,

$$f_h(x)=f_r(x) \quad (x\neq c) \quad (11)$$

$$f_d(x)=0 \quad (x\neq c) \quad (12)$$

Left continuous component $f_h(x)$ of formula (8) is continuous in interval $x\neq c$, and is pieswise continuous function. Step component $f_d(x)$ of formula(9) is discrete function.

(5) Expression by discrete function

When mass distribution of a stone is expressed by function $f_1(x)$, as mass of magnitude P is concentrated at point $x=d$, formula(13) holds.

$$f_1(d)=P \quad (13)$$

As there is no stone in interval $x\neq d$, formula(14) holds.

$$f_1(x)=0 \quad (x\neq d) \quad (14)$$

Mass distribution $f_1(x)$ of stone is expressed by discrete function of formula (13) and formula(14).

(6) Unification of mass of pencil and stone

Suppose function $f(x)$ which unifies mass distribution of pencil and stone. Function $f(x)$ is superposition of function $f_0(x)$ and function $f_1(x)$, but is not simple addition. Attaching unit, superposition is expressed as formula(15).

$$f(x)\text{kg/m}=f_0(x)\text{kg/m}+f_1(x)\text{kg} \quad (15)$$

Supposing that unit kg/m be major, function $f(x)$ is also attached unit kg/m. Dividing both sides of formula(15) by unit kg/m, formula(16) is obtained.

$$f(x)=f_0(x)+f_1(x)\cdot m \quad (16)$$

In formula(16), unit m is the mark which distinguishes component $f_0(x)$ and component $f_1(x)$. $f_0(x)$ of formula(16) is called 0-th component, Component $f_1(x)$ of formula(16) is called 1st component. When the form of numerical vector is used, formula(16) is altered to formula(17).

$$f(x)=\{f_0(x),f_1(x)\} \quad (17)$$

Substitute formula(10) into formula(17), formula(18) is obtained.

$$f(x)=\{f_h(x),f_d(x),f_1(x)\} \quad (18)$$

Mass distribution is expressed by function of 3 components which are left continuous component, step component, 1st component.