Introduction from route location to hyperfunction

2nd Nov. 2013

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(1) Introduction

To differentiate indifferentiable function forcibly is one of the purpose of hyperfunction theory. In the process of route location in civil engineering, a curve is inserted into polygonal line which is not smooth. Then the polygonal line becomes smooth. Suggested from route location, the introduction part of hyper-function theory is tried to compose.

(2) Polygonal line

In the process of road or railway design, route location is carried out. Considering the condition of terrain and land use, lines are sequenced from the starting point to end point. At first, shown as Fig-1, line AB, line BC, line CD,

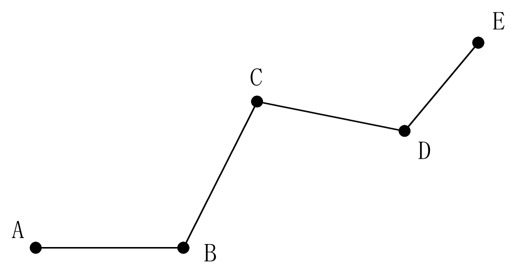

Fig-1 Polygonal line which is sequenced lines

line DE are connected in sequence from the starting point A to end point E. It is bended at point B, point C, point D, and is not smooth. It is a polygonal line which is composed of sequenced lines.

(3) Curve insertion

Taking the ABC part from Fig-1 out and enlarging, Fig-2 is drawn. Near point B

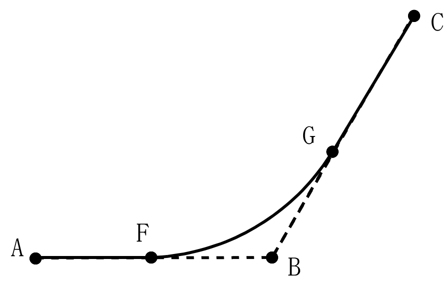

Fig-2 Connected lines and arc

where polygonal line is bended, inserting arc FG, line AB and line BC are connected smoothly. Formula(1) holeds.

$$BF = BG \tag{1}$$

In case arc FG is inserted, force perpen-dicular to moving direction does not act on a vehicle moving along straight line AF. Because of centrifugal force, however, force which is perpendicular to moving direction does act on a vehicle moving alon g arc part FG. In case a vehicle move along route AFGC, if radius of arc FG be r, the curvature discontinuously increases suddenly from 0 to $\frac{1}{r}$ at point F. If speed is constant, as centrifugal force is pro-portional to speed, centrifugal force in-creases suddenly at point F, and harms the ride quality. It is dangerous.

(4) Transition curve

To avoid sudden increase of centrifugal force, shown as Fig-3, part JK of arc FG is remaining, from point H which is nearer point A than point F to point J, curvature

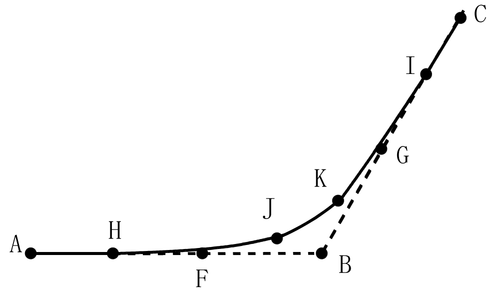

Fig-3 Insertion of trnsition curve

is changed smoothly. Curve HJ is called transition curve. Curve KI is also transition curve. When road is designed, clothoid curve is often used as transition curve.

(5) Approximate function

Function f(x) expressed by foumula(2), foumula(3) is bended at point x=0, and is indefferentiable.

$$f(x) = 0 \qquad (x \leqq 0) \qquad (2)$$

$$f(x) = x \qquad (0 \leqq x) \qquad (3)$$

Function F(x) by formula(4), formula(5), foumula(6) which is inserted transition curve in interval $-\varepsilon \leqq x \leqq +\varepsilon$, is smooth within whole area of real number, and is differentiable.

$$F(x) = 0 \qquad (x \leqq -\varepsilon) \qquad (4)$$

$$F(x) = \frac{1}{32\varepsilon^5}x^6 - \frac{5}{32\varepsilon^3}x^4 + \frac{15}{32\varepsilon}x^2 + \frac{1}{2}x + \frac{5}{32}\varepsilon \qquad (-\varepsilon \leqq x \leqq +\varepsilon) \qquad (5)$$

$$F(x) = x \qquad (+\varepsilon \leqq x) \qquad (6)$$

Consiering limit of parameter $\varepsilon \to 0$, as formula(7) holds, function F(x) is approximate function of function f(x).

$$\lim_{\varepsilon \to 0} F(x) = f(x) \qquad (7)$$

The relation between function f(x) by foumula(2), foumula(3) and function F(x) by formula(4), formula(5), foumula(6) is similar to the relation between polygonal line AHFBGIC shown in Fig-3 and curve AHJKIC shown in Fig-3. Regarding the lines by formula(4), foumula(6) by means of inserting curve by formula(5) within interval $-\varepsilon \leqq x \leqq +\varepsilon$, lines are connected smoothly.

(6) Contradictory 2 characteristics

Function f(x) is indifferentiable at point x=0, however, function F(x) is differntiable by means of formula(5). The "thing" which has contradictory characteristics of combined differentiable and indiffernṫiable, is called hyper-function f(x). If necessary, they are distinguished by adding note such as function f(x) or hyperfunction f(x).

Being not completely finished limit operation of formula(7), parameter ε is fixed to small constant. Because value ε is constant, function F(x) is differntiable. Because value ε is small, interval $-\varepsilon \leqq x \leqq +\varepsilon$ can be thought to be the same as point x=0, and can be thought that formula (8) may hold.

$$F(x) \fallingdotseq \text{function } f(x) \qquad (8)$$

The state of formula(8) is the hyper-function f(x).

Function F'(x) is approximate function of Heaviside function $\eta(x)$. Function F''(x) is approximate function of Dirac function $\delta(x)$.

(7) Conclusion

The indifferentiable part of a function can be changed to differentiable, by inserting smooth curve. Function which is changed to differentiable by inserting curve is called hyperfunction. Inserting curve into function is similar to inserting curve in the process of route location.