Discovery of microdomain radius parameter

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(1) Approximation by normal function

Function $\Delta(x)$ expressed by formula(1) is normal distribution function which has value 0 as mean value, value $\frac{\varepsilon}{\sqrt{2}}$ as standard deviate.

$$\Delta(x) = \frac{1}{\varepsilon\sqrt{\pi}} \exp(-(\frac{x}{\varepsilon})^2) \qquad (1)$$

Function $\Delta(x)$ is often used as approximate function of Dirac function $\delta(x)$.

(2) Concentration of definite integral

Dirac function $\delta(x)$ describes the situation that quantity of magnitude 1 is concentrated at the point x＝0. It fits to description of integral expressed by formula(2).

$$\int_{-0}^{+0} \delta(x)\,dx = 1 \qquad (2)$$

The right side of the formula(2) shows the magnitude 1 and integral interval $-0 \leqq x \leqq +0$ of left side shows the point x＝0. Formula(2) means the integral at point x＝0. Formula(2) suggests the integral at point x＝0 is a value other than 0, and is called concentration of definite integral. But the case rarely be seen in which formula(2) is used to explain Dirac function $\delta(x)$. Frequently formula(3) is used to explain Dirac function $\delta(x)$.

$$\int_{-\infty}^{+\infty} \delta(x)\,dx = 1 \qquad (3)$$

The right side of formula(3) shows the magnitude 1, but the integral interval of left side spans whole range of real number $-\infty < x < +\infty$ and deos not suggest point x＝0. Formula(3) may be recognized the quantity of magnitude 1 is dispersed along interval of $-\infty < x < +\infty$. Not formula(3) but formula (2) is suitable in order to describe concentration of definite integral.

(3) Calculation using approximate function

Interval $-\varepsilon \leqq x \leqq +\varepsilon$ converges interval $-0 \leqq x \leqq +0$ when limit of parameter $\varepsilon \to 0$, therefore it is expected that formula(4) would hold and formula(2) would be gotten.

$$\lim_{\varepsilon \to 0} \int_{-\varepsilon}^{+\varepsilon} \Delta(x)\,dx = 1 \qquad \text{(Not holds)} \qquad (4)$$

But, formula(4) does not hold. Calculate formula(5) in order to calculate formula(4).

$$\int_{-\varepsilon}^{+\varepsilon} \Delta(x)\,dx = \int_{-\varepsilon}^{+\varepsilon} \frac{1}{\varepsilon\sqrt{\pi}} \exp(-(\frac{x}{\varepsilon})^2)\,dx \qquad (5)$$

Under variable transformation of formula(6), formula(7) holds.

$$\frac{x}{\varepsilon} = \frac{z}{\sqrt{2}} \qquad (6)$$

$$dx = \frac{\varepsilon}{\sqrt{2}}\,dz \qquad (7)$$

As $x = -\varepsilon$ corresponds $z = -\sqrt{2}$ and $x = +\varepsilon$ corresponds $z = +\sqrt{2}$, formula(8) is calculated.

$$\begin{aligned}\int_{-\varepsilon}^{+\varepsilon} \Delta(x)\,dx &= \int_{-\sqrt{2}}^{+\sqrt{2}} \frac{1}{\varepsilon\sqrt{\pi}} \exp(-\frac{z^2}{2}) \frac{\varepsilon}{\sqrt{2}}\,dz \\ &= \int_{-\sqrt{2}}^{+\sqrt{2}} \frac{1}{\sqrt{2\pi}} \exp(-\frac{z^2}{2})\,dz \\ &= 2\int_{0}^{+\sqrt{2}} \frac{1}{\sqrt{2\pi}} \exp(-\frac{z^2}{2})\,dz \qquad (8)\end{aligned}$$

As the integrand on the last hand side of formula(8) is normal distribution function with mean value 0 and standard deviation 1, letting $\sqrt{2}$＝1.41 get 0.4207 from function table, formula(9) is calculated.

$$\int_{-\varepsilon}^{+\varepsilon} \Delta(x)\,dx = 2\times 0.4207 = 0.841 \qquad (9)$$

Even when limit of $\varepsilon \to 0$, formula(4) can not be obtained from formula(9). As formula (10) holds using function $\Delta(x)$ expressed by formula(1), formula(3) certainly holds.

$$\lim_{\varepsilon\to 0} \int_{-\infty}^{+\infty} \Delta(x)\,dx = 1 \qquad (10)$$

(4) Separation into two parameters

As for parameter ε in approximate function $\Delta(x)$ by formula(1), when limit of $\varepsilon \to 0$, function value $\Delta(0)$ at point $x=0$ diverges then the point $x=0$ be singular point. Parameter ε is called singularizing parameter. Introducing parameter ρ which is different from ε, formula(11) holds.

$$\lim_{\rho\to 0} \lim_{\varepsilon\to 0} \int_{-\rho}^{+\rho} \Delta(x)\,dx = 1 \qquad (11)$$

It is reasonable that formula(11) means formula(2). As this parameter ρ is the radius of interval $-\rho \leqq x \leqq +\rho$ which is supposed to be the same as point $x=0$, parameter ρ is called microdomain radius parameter. The explanation by formula (11) is created by the author in which microdomain radius parameter is used. As far as formula (11) is concerned it is important that singularizing parameter varies to limit faster than microdomain radius parameter ρ. In formula(9) compared to formula (11), singularizing parameter and microdomain radius parameter are not yet separated and are used dual purpose.

Transform expressed by formula(12), formula(13) is obtained.

$$\frac{x}{\varepsilon} = y \qquad (12)$$

$$\frac{dx}{\varepsilon} = dy \qquad (13)$$

As ε varies to limit faster than ρ, when $\varepsilon \to 0$, $x=-\rho$ corresponds $y=-\infty$, $x=+\rho$ corresponds $y=+\infty$, formula(14) is calculated, then formula(11) holds.

$$\int_{-\rho}^{+\rho} \frac{1}{\varepsilon\sqrt{\pi}} \exp\left(-\left(\frac{x}{\varepsilon}\right)^2\right) dx \to \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \exp(-y^2)\,dy = 1 \qquad (14)$$

(5) Idea of component

As for without singular point $x=0$, assume formula(15) suggested from formula (11).

$$\delta_1(x) = \lim_{\rho\to 0} \lim_{\varepsilon\to 0} \int_{x-\rho}^{x+\rho} \Delta(t)\,dt \qquad (15)$$

If let $x=0$ on right side of formula(15), and change variable t to variable x, left side of formula(11) is obtained. When formula(15) is calculateed , formula(16), formula(17) are obtained.

$$\delta_1(x) = 1 \qquad (x=0) \qquad (16)$$

$$\delta_1(x) = 0 \qquad (x\neq 0) \qquad (17)$$

Function $\delta_1(x)$ of formula(16), formula(17) is characteristic function which explains Dirac function $\delta(x)$ and is called first degree concentration component.

(6) Importance of the discovery

Before the idea to separate microdomain radius parameter ρ from singularizing parameter ε is conceived, formula(11) could not be calculated, nor concentration of definite integral could not be explained using formula(2). Therefore, formula(10) was calculated instead and formula(3) had to be used to explain. Because microdomain radius parameter is discovered, formula(3) become no need to explain.