# Hyperfunction as the expanded concept of function

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## (1) Expansion of a concept

For example, natural number can be carried out addition, and subtraction is reverse operation of addition, but subtraction can not always be carried out. Addition expressed by formula(1), derives subtraction expressed by formula(2).

$2+3=5$ (1)

$5-3=2$ (2)

Formula(1) and formula(2) are the reverse operation. Subtraction expressed by formula(3) does no have solution within natural number.

$2-3$ (without natural number) (3)

If 0 and negative number is added to natural number, subtraction expressed by formula(3) does have a solution as expressed by formula(4).

$2-3=-1$ (4)

Natural number added 0 and negative number is called integer. The procedure to obtain integer from natural number is a expansion of concept of number.

Integer added fraction is called rational number. Rational number added irrational number is called real number. Real number added imaginary number is called complex number. These all procedures are expansion of concept of number.

## (2) Main nature of concept

When integer is obtained from natural number, there is a nature lost. There exists the first number 1, and the number 1 is the smallest number of the natural number. There are many numbers smaller than 1, such as 0、-1、-2、・・・、and the first number, the smallest number does not exist. When the expansion is carried out, although the nature that there exist first number is lost, the nature is thought not to be the main nature,

As for natural nunber, the nature that subtraction cn be carried out is not perfect. Subtraction expressed by formula (3) can not be answered within natural number. When integer is derived from natural number, the nature that subtraction can be carried out is chosen as the main nature, and is fully enforced.

When expansion of concept is carried out, the main nature is not be lost.

## (3) Complaint about existing theory

As for hyperfunction, ″distribution by Schwartz″ or ″hyperfunction by Satou″ are existing. Although theory by Schwartz or theory by Satou are exact, the author does have complaint because he tries to understand hyperfuction as the concept expansion of function.

Correspondence from independent variable into dependent variable is the fundamental definition of function. The points where the correspondence does not exist, are out of domain, and out of domain is out of discussion, out of consideration. Correspondence is the main nature of function concept, this main nature should not be lost, when concept expansion is carried out. Even for hyperfunction, domain should be clearly specified, and function value of each point within the domain should be defined.

## (4) Singular point of Dirac function

The function value of Dirac function does not exist at the point x=0. The point must be out of domain, because function value does not exist. Under normal conditions, out of domain is out of discussion, out of consideration. As far as Dirac function is concerned, if the point x=0 be kept out of domain, the theory of hyperfunction does not have any meaning. In order to keep the point x=0 be within domain, the author would like to define function value. Because expansion of concept is carried out, function value of not ordinary or exact meaning might be accepted. Although the author would like to explain that a quantity of magnitude 1 is concentrated at the point x=0 of Dirac function, it is not suitable to express simply by formula(5), formula(6).

$$\delta(x)=1 \quad (x=0) \quad \text{(not suitable)} \qquad (5)$$

$$\delta(x)=0 \quad (x\neq 0) \quad \text{(not suitable)} \qquad (6)$$

(5) Introduction of component expression

In order to explain the characteristic of singular point of hyperfunction, component is introduced as an expression of hyperfunction. Defining left continuous component $f_h(x)$, step component $f_d(x)$, first degree component $f_1(x)$, second degree component $f_2(x)$, ･･･, lining up components, hyperfunction f(x) is expressed by formula (7).

$$f(x)=\{f_h(x), f_d(x), f_1(x), f_2(x), \cdots\} \qquad (7)$$

Using microdomain radius parameter $\rho$ and approximate function F(x) which contains singularizing parameter $\varepsilon$, components of hyperfunction f(x) are calculated by formula(8)～formula(10).

$$f_h(x)=\lim_{\rho\to 0}\lim_{\varepsilon\to 0} F(x-\rho) \qquad (8)$$

$$f_d(x)=\lim_{\rho\to 0}\lim_{\varepsilon\to 0} \{F(x+\rho)-F(x-\rho)\} \qquad (9)$$

$$f_n(x)=\lim_{\rho\to 0}\lim_{\varepsilon\to 0}\int_{x-\rho}^{x+\rho}(t-x)^{n-1}F(t)\,dt$$

$$(n=1、2、3、\cdot\cdot\cdot) \qquad (10)$$

At regular point, components without left continuous component $f_h(x)$ are all constant function with function value 0.

Function $\Delta(x)$ expressed by formula(11) is one of approximate functions of Dirac function $\delta(x)$.

$$\Delta(x)=\frac{1}{\varepsilon\sqrt{\pi}}\exp\left(-\left(\frac{x}{\varepsilon}\right)^2\right) \qquad (11)$$

Substituting $\Delta(x)$ of formula(11) into formula(8)～formula(10) as the approximate function F(x), components $\delta_h(x)$, $\delta_d(x)$, $\delta_1(x)$, $\delta_2(x)$, ･･･ are calculated. Then substituting components into formula(7), Dirac function $\delta(x)$ is expressed by formula (12), formula(13).

$$\delta(x)=(0, 0, 1, 0, 0, \cdot\cdot\cdot) \quad (x=0) \qquad (12)$$

$$\delta(x)=(0, 0, 0, 0, 0, \cdot\cdot\cdot) \quad (x\neq 0) \qquad (13)$$

Formula(12) is the function value of Dirac function $\delta(x)$ at singular point. First degree component $\delta_1(x)$ expressed by formula (12), formula(13) is the same as function $\delta(x)$ expressed by formula(5), formula(6).

(6) Internal variation of a point

The gist of theory of component type hyperfunction is the idea that explains the characteristic of each point from internal variation of the point. Interval $x-\rho \leqq t \leqq x+\rho$ of approximate function F(x) is called microdomain at point t=x of hyperfunction f(x). Variation of approximate function in the microdomain is called internal variation at a point of hyperfunction f(x). Intense internal variation builds up the characteristic of singular point. $\Delta(0)$ of formula(11) diverges. Internal variation at regular point is not intense.