Chapter 1 Expression of Load Distribution

(1) 3 kinds of load

3 kinds of load, that is distributed load, concentrated load, concentrated moment be applied on a structure. Fig.1-1 shows a simple beam which is ℓ

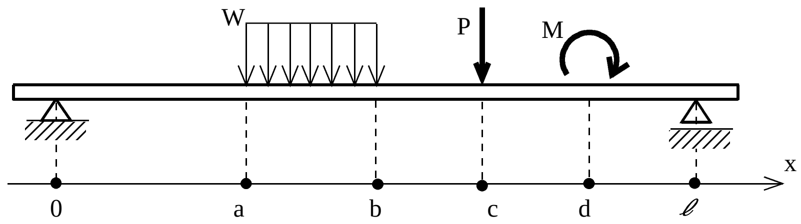

Fig.1-1 3 kinds of load

long and the member axis represents the beam. Because load is distributed at each point along the member axis, it is natural that the function which independent variable is coordinate x set along the member axis, be used in order to express the load. The state of load at each point of member axis is unique. Setting a coordinate x along the member axis, and assume that left edge $x=0$, right edge $x=\ell$. Uniformly distributed load which magnitude is W is applied from point $x=a$ to point $x=b$, concentrated load P is applied at point $x=c$, concentrated moment M is applied at point $x=d$.

Uniformly distributed load has unit N/m, and is expressed by many parallel fine arrow shown like Fig.1-1. Concentrated load has unit N, and is expressed by a thick arrow shown like Fig.1-1, concentrated moment has unit Nm, and is expressed by a thick arrow arc shown like Fig.1-1. When we express load using function, as far as the simple beam of Fig.1-1 is concerned, the part between the 2 supports is our discussion object, so the interval $0 \leqq x \leqq \ell$ is the domain of definition. Although it is natural that load is expressed using function, load can not be expressed using one function, as different kinds of load is applied. Different kinds of load be expressed by

different functions, dividing components, and we should discuss about common characteristic of load.

(2) Distributed load

[Example]

As an example of uniformly distributed load of Fig.1-1, suppose a load by a parallelepiped solid ABCD shown as upper part of Fig.1-2. Set coordinate

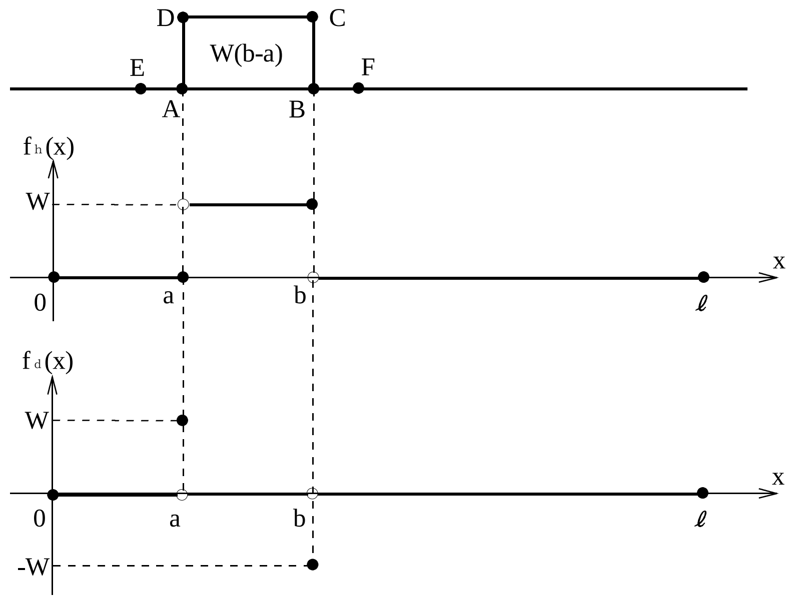

Fig.1-2 Parallelepiped solid by outline view

x along the road surface. In order to simplify our discussion the width of road surface and the width of parallelepiped solid be the same, and the length of parallelepiped be from the point A coordinate $x=a$ to the point B coordinate $x=b$ along the road surface. Weight W(b-a) of parallelepiped applies upon road surface as load.

[Component expression]

Although distributed load is expressed by continuous function in the interval $0 \leqq x < a$ or interval $a < x < b$ or interval $b < x \leqq \ell$, as the function value increase suddenly at the point $x = a$ and decrease suddenly at the point $x = b$, the load can not be expressed by ordinary function. Let's express distributed load $f_o(x)$ by dividing it into left continuous component $f_h(x)$ of Formula1･1, Formula1･2 and step component $f_d(x)$ of Formula1･3, Formula 1･4, Formula1･5.

$f_h(x) = 0 \qquad (0 \leqq x \leqq a、b < x \leqq \ell) \qquad$ 1･1

$f_h(x) = W \qquad (a < x \leqq b) \qquad$ 1･2

$f_d(x) = 0 \qquad (0 \leqq x < a、a < x < b、b < x \leqq \ell) \qquad$ 1･3

$f_d(x) = W \qquad (x = a) \qquad$ 1･4

$f_d(x) = -W \qquad (x = b) \qquad$ 1･5

Domain of function $f_h(x)$ and function $f_d(x)$ is interval $0 \leqq x \leqq \ell$. The unit of left continuous component $f_h(x)$ and step component $f_d(x)$ are the same N/m. Left continuous component $f_h(x)$ is shown as middle part of Fig.1-2, step component $f_d(x)$ is shown as lower part of Fig.1-2. At the point $x = b$, the minus value of step conponent $f_d(x)$ expresses sudden decreas. Distributed load $f_o(x)$ is a set of left continuous component $f_h(x)$ and step component $f_d(x)$, so it may be expressed by demarcated by comma , wrapped up by brancket { } like Formula1･6, same form as numerical vector.

$f_o(x) = \{f_h(x), f_d(x)\} \qquad$ 1･6

If we would express it same form as vector, we could express it like Formula 1･7 using numerical value 1 and symbol $\uparrow$ as basis vector.

$f_o(x) = f_h(x) + f_d(x)\uparrow \qquad$ 1･7

The symbol $\uparrow$ is called step unit. The symbol $\uparrow$ is made by imitating the step form that function $f_h(x)$ increases suddenly near point $x = a$, and showing direction of increase by arrow. Altough we can use capital letter "ダ" or "d" as a symbol of step unit, capital letter "ダ" is from word "ダンサ" in Japanese letter, capital ltter "d" is from Japanese word "dansa" in Latain letter, we chose $\uparrow$ in this book. Although the unit of left continuous

component $f_h(x)$ and step component $f_d(x)$ are the same N/m, their characteristics are different, they are shown as different components.

[Point not determined uniquely]

Distributed load $f_o(x)$ is expressed as Formula1･6 or Formula1･7 in order to determine the dependent variable of function $f_o(x)$ uniquely. If we try to express distributed load by ordinary function $f_z(x)$, as far as interval $0 \leqq x < a$, interval $a < x < b$, interval $b < x \leqq \ell$ are concerned we can express like Formula1･8 and Formula1･9, but at point $x = a$ and pointx$= b$, function value is discontinuous, we can not determine function value $f_z(a)$, $f_z(b)$ uniquely.

$f_z(x) = 0 \qquad (0 \leqq x < a、b < x \leqq \ell) \qquad$ 1･8

$f_z(x) = W \qquad (a < x < b) \qquad$ 1･9

Function forms steps at point $x = a$ and point $x = b$. If we suppose that $f_z(a) = 0$, $f_z(b) = W$, or $f_z(a) = W$, $f_z(b) = 0$, or $f_z(a) = \frac{W}{2}$, $f_z(b) = \frac{W}{2}$, or point $x = a$ and $x = b$ are out of domain of the function, we can not express properly the state of step of distributed load. In hitherto existing textbook[1)] distributed load of parallelepiped solid shown as upper part of Fig.1-2, is expressed by function shown as Formula1･8 and Formula1･9, and it is avoided to mention the state at point $x = a$ and $x = b$. As the state of load at each point along member axis is unique, it is necessary to determine the value of dependent variable corresponding to each value of independent variable. Function value $f_z(a)$, $f_z(b)$ of Formula1･8 and Formula1･9 are not determined uniquely, therefore the state at point $x = a$ and $x = b$ should be explained thoroughly, by expressing distributed load of $f_o(x)$ as a set of functions like Formula1･6 or Formula1･7, determined the value of dependent variable uniquely. Step component $f_d(x)$ shown as Formula1･3, Formula1･4, Formula1･5 is component to express sudden increase at point $x = a$ or sudden decrease at point $x = b$.

[Detail gaze]

When we approach seeing Fig.1-2 and gaze at the part of EF magnifying in detail, it becomes to look like the upper part of Fig.1-3. Parallelepiped solid ABCD deformed to be GHIJKLCD, road surface straight line EF

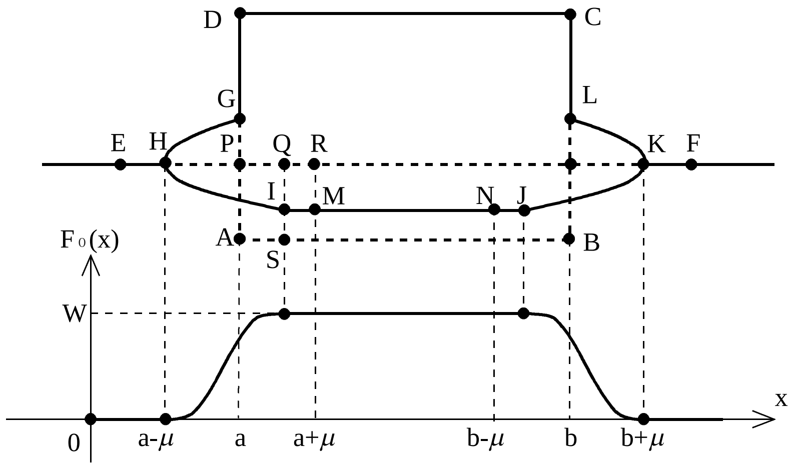

Fig.1-3　Parallelepiped solid by detailed gaze

deformed to be curve EHIJKF, and weight W(b-a) of parallelepiped transmitted to road surface through the curve HIJK. A rough sketch of load $F_0(x)$ transmited is shown as as the lower part of Fig.1-3. The unit of function $F_0(x)$ is N/m, and is the same as the unit of function $f_0(x)$. The part near point A is deformed complicated, and distributed load $F_0(x)$ increases from 0 to W smoothly within the curve HI. As far as Fig.1-3 is concerned, the length HP is longer than the length PQ, the longer length HP be μ, choose point R where the length PR be μ, we suppose straight line HR instead of straight line HQ. As the interval $a-\mu \leqq x \leqq a+\mu$ expresses straight line HR, distributed load $F_0(x)$ increases from 0 to W in the interval $a-\mu \leqq x \leqq a+\mu$ as shown in the lower part of Fig.1-3, Formula1・10, Formula 1・11 hold.

$$f_h(a) = F_0(a-\mu) \qquad 1 \cdot 10$$

$$f_d(a) = F_0(a+\mu) - F_0(a-\mu) \qquad 1 \cdot 11$$

Distributed load $F_0(x)$ decreases from W to 0 near point B likewise as near point A. If the quality of material of parallelepiped and road surface are uniform, we also may suppose length μ near point B. Uniform distributed load applies in the part of straight line IJ, we suppose straight line MN instead of straight line IJ, function $F_0(x)$ is expressed by Formula 1・12.

$$F_0(x) = W \qquad (a+\mu \leqq x \leqq b-\mu) \qquad 1 \cdot 12$$

The whole weight of parallelepiped solid transmitted through interval $a-\mu \leqq x \leqq b+\mu$, and is the same as the total force W(b-a), so Formula1・13 holds.

$$\int_{a-\mu}^{b+\mu} F_0(x)dx = \int_{a+0}^{b} f_h(x)dx = W(b-a) \qquad 1 \cdot 13$$

Middle hand side of Formula1・13 means comprehensive integral of Formula 1・14.

$$\int_{a+0}^{b} f_h(x)dx = \lim_{\varepsilon \to 0} \int_{a+\varepsilon}^{b} f_h(x)dx \qquad 1 \cdot 14$$

As left continuous component $f_h(x)$ of distributed load is right inintegrable at the point $x = a$, comprehensive integral should be used. Let ε be plus number, point $x = a+\varepsilon$ is integrable, and right hand side of Formula1・14 converges. Left continuous component of distributed load is left integrable at the point $x = b$. Function $F_0(x)$ is continuous and integrable in the interval $a-\mu \leqq x \leqq b+\mu$.

[Equivalent detail function]

In the upper part of Fig.1-3, if the stifness of road surface and parallelepiped changes curve HIJK also changes. IS is deformation amount of parallelepiped, and QI is deformation amount of road surface, QS is the total deformation amount. If the stifness of parallelepiped is high and the stifness of road surface is low, IS is small and QI is large. Inversely if the stifness of parallelepiped is low and the stifness of road surface is high, IS is large and QI is small. Both together are large, QS is small, inversely both are small together, QS is large. If curve HIJK differs, detail function $F_0(x)$ also differs, even if outline function $f_0(x)$ is the same, detail function $F_0(x)$ is different. Correspondence between function $f_0(x)$ and function $F_0(x)$ is one to many. When different detail function $F_0(x)$ correspond to the same outline function $f_0(x)$, we would consider the different detail function $F_0(x)$ are equivalent each other.

[Outline View]

When we back away seeing Fig.1-3, 9 points G,H,I,M,A,S,P,Q,R become to overlap and not to be distinguished gradually, it becomes look like point A of the upper part of Fig.1-2. As Fig.1-2 sees wide range from a distance, it is the state of outline view. As Fig.1-3 observes each part in detail, it is the state of detail gaze. The auther proposes that the difference between outline view and detail gaze be called sight, and that handling the 2 sight be called sight transfer. As distrbuted load is understood by a pair of function $f_0(x)$ and $F_0(x)$, the auther proposes that it be called bisight function, function $f_0(x)$ be called outline function, function $F_0(x)$ be called detail function. As "point $x=a$ of Fig.1-2" and "interval $a-\mu \leqq x \leqq a+\mu$ of Fig.1-3" corresponds by sight transfer, the author proposes that point $x=a$ of Fig.1-2 be called microdomain point, interval $a-\mu \leqq x \leqq a+\mu$ of Fig.1-3 be called microdomain, length μ be called microdomain radius. Microdomain radius μ is very small plus number. Microdomain radius μ is attached to the gaze function $F_0(x)$, there exist different microdomain radius μ for each of many different equivalent function $F_0(x)$. Point $x=b$ of Fig.1-2 is also microdomain point. If the quality of material of parallelepiped and road surface are uniform, microdomain radius at point $x=b$ of Fig.1-2 is also the same μ as at point $x=a$. A point which is not microdomain point is called usually point.

[Domain of definition]

Edge points $x=a$ and $x=b$ of partially loaded uniformly distributed load in Fig.1-1 are microdomain points, and point of application $x=c$ of concentrated load and point of application $x=d$ of concentrated moment are also microdomain points. Points $x=0$ and $x=\ell$ are supports, where upward

concentrated force act as reaction forces. Points $x=0$ and $x=\ell$ where reaction forces act are also microdomain points. Considering microdomain $-\alpha \leqq x \leqq +\alpha$ with microdomain radius α and microdomain point $x=0$, microdomain $\ell-\beta \leqq x \leqq \ell+\beta$ with microdomain radius β and microdomain point $x=\ell$, domain of definition of detail function $F_0(x)$ is interval $-\alpha \leqq x \leqq \ell+\beta$, which corresponds interval $0 \leqq x \leqq \ell$ which is domain of definition of outline function $f_0(x)$. Fig.1-4 shows a illustrated log bridge. A log is supported at both banks the weight of a log and a man is transmitted to the banks. The log is not just the same as the width of the river as shown

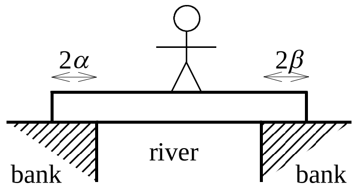

Fig.1-4　Possible log bridge

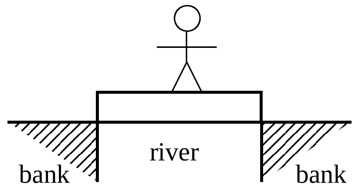

Fig1-5　Inpossible log bridge

in Fig.1-5, but a little bit longer as shown in Fig.1-4. If a log bridge does not overlap with banks of about a diameter of the log, the weight of the log and a man can not be tarnsmitted to the banks. The log overlaps with the left bank of length 2α, the log overlaps with the right bank of length 2β, α and β of Fig.1-4 are microdomain radiuses. Fig.1-4 and Fig.1-5 shows that a support is a microdomain point. Interval $0<x<\ell$ of outline function $f_0(x)$ corresponds interval $+\alpha \leqq x \leqq \ell-\beta$ of detail function $F_0(x)$. Out of interval $a-\mu \leqq x \leqq b+\mu$, distributed load is not transmitted, Formula1・15 holds.

$$F_0(x)=0 \qquad (-\alpha \leqq x \leqq a-\mu,\ b+\mu \leqq x \leqq \ell+\beta) \qquad 1\cdot 15$$

Detail function $F_0(x)$ satisfies Formula1・12、Formula1・13、Formula1・15.

[Distribution without step]

Load shown in Fig.1-2, Fig.1-3 has step at point A and B, we suppose distributed load without any step. The upper part of Fig.1-6 shows a state that sand is heaped up on the road surface CD from point A to point B. As

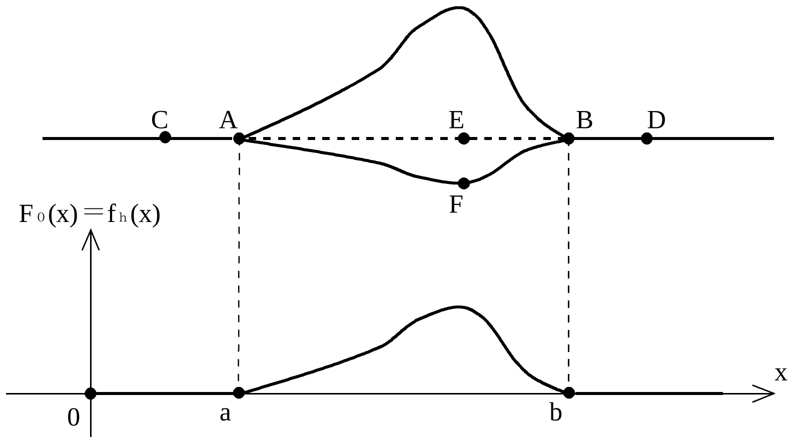

Fig. 1-6 Distributed load without step

sand is not viscous, even at point A and point B the shape is continuous without step. Road surface straight line CD deformed to curve CAFBD, load is transmitted through the curve AFB. A rough sketch of load taransmitted is shown as lower part of Fig.1-6. Distributed load shown in Fig.1-6 does not have any microdomain point, and is continuous in all of domain of definition, and Formula1・16 holds.

$f_d(x)=0 \qquad (0 \leqq x \leqq \ell) \qquad 1\cdot16$

As far as distributed load shown in Fig.1-3 and distributed load shown in Fig.1-6 are concerned, at ordinary point Formula1・17 holds.

$F_0(x)=f_h(x) \qquad$ (x is ordinary point) $\qquad 1\cdot17$

(3) Concentrated load

[Example]

As an example of concentrated load of Fig.1-1, we suppose wheel load of a car of upper part of Fig.1-7. We suppose the state that rear wheel touches road surface BC at point A, and we do not suppose front wheel D. Load P

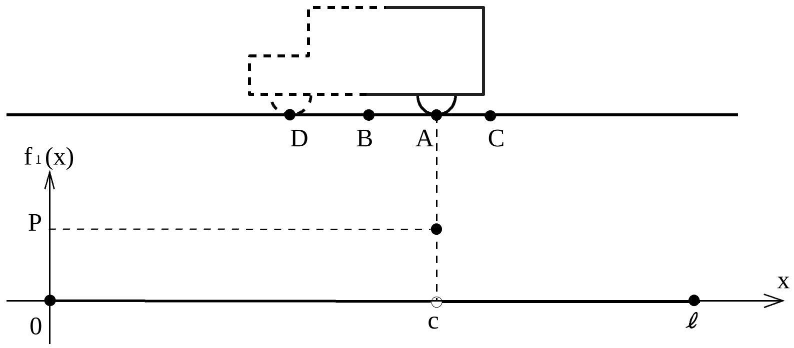

Fig.1-7 Wheel load by outline view

which is alloted to rear wheel among car weight, is transmitted from wheel to road surface. Set coordinate of x along the road surface, coordinate of point A be x＝c. Let function $f_1(x)$ express load, it is shown in lower part of Fig.1-7, and expressed by Formula1・18 and Formula1・19.

$f_1(x)=0 \qquad (0 \leqq x < c、c < x \leqq \ell) \qquad 1\cdot18$

$f_1(x)=P \qquad (x=c) \qquad 1\cdot19$

The unit of function $f_1(x)$ is the same N as the unit of load P. Function $f_1(x)$ is outline function and its domain of definition is interval $0 \leqq x \leqq \ell$.

[Detail gaze]

When we approach seeing Fig.1-7 and gaze at the part of BC magnifying in detail, it becomes to look like the upper part of Fig.1-8. As for the wheel which center is E, curve FAG deformed to curve FIG, as for road surface strait line BC deformed to curve BFIGC, wheel load is transmitted to road surface through curve FIG. As wheel is symmetry with respect to line AE, FH＝HG, this length be ν. Weel load is distributed force in microdomain $c-\nu \leqq x \leqq c+\nu$, rough sketch is shown as the function $F_1(x)$ of the lower part of Fig.1-8. The unit of function $F_1(x)$ is N/m, and function $f_1(x)$ and function $F_1(x)$ is different in unit. Function $F_1(x)$ is detail function, and its domain of definition is interval $-\alpha \leqq x \leqq \ell+\beta$. As distrbuted force $F_1(x)$ is not distrbuted

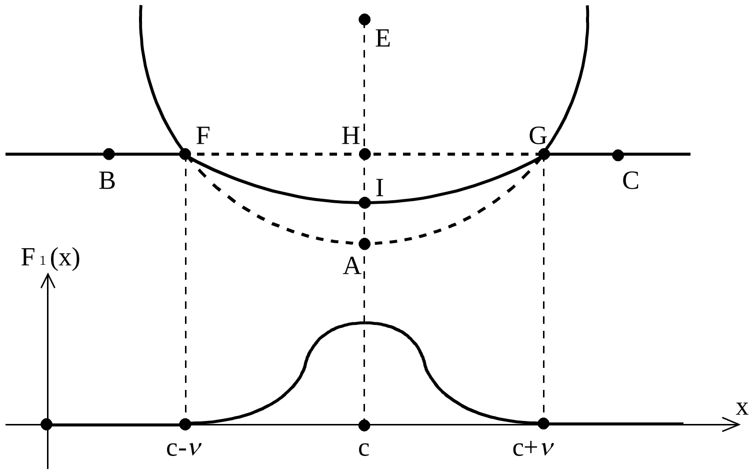

Fig.1-8 Wheel load by detail gaze

out of microdomain $c-\nu \leqq x \leqq c+\nu$ Formula1・20 hold.

$$F_1(x)=0 \qquad (-\alpha \leqq x \leqq c-\nu,\ c+\nu \leqq x \leqq \ell+\beta) \qquad 1\cdot 20$$

As total force of distributed force $F_1(x)$ is same as the value of P of function $f_1(x)$ at point x=c, Formula1・21 holds.

$$\int_{c-\nu}^{c+\nu} F_1(x)dx = f_1(c) = P \qquad 1\cdot 21$$

As moment of distributed force $F_1(x)$ about the point $x=c$ is 0, Formula 1・22 holds.

$$\int_{c-\nu}^{c+\nu} F_1(x)(x-c)dx = 0 \qquad 1\cdot 22$$

[Equivlent detail function]

In the upper part of Fig.1-8, if the stifness of road surface and wheel changes, curve FIG also changes. AI is deformation amount of wheel, HI is deformation amount of road surface, AH is the total deformation amount. If the stifness of wheel is high and the stifness of road surface is low, AI is small and HI is large. Inversely if the stifness of wheel is low and the stifness of road surface is high, AI is large and HI small. Both together are large, AH is small, inversely both are small together, AH is large. If curve FIG differs detail function $F_1(x)$ also differs, even if outline function $f_1(x)$ is the same. Correspondence between outline function $f_1(x)$ and detail function $F_1(x)$ is one to many. When different detail functions $F_1(x)$ correspond to the same outline function $f_1(x)$, we would consider the differet detail function $F_1(x)$ are equivalent each other.

[Outline view]

When we back away seeing Fig.1-8, 5 points A, F, G, H, I become to overlap and not to be distingushed gradually, it becomes look like point A of the upper part of Fig.1-7. As "point $x=c$ of Fig.1-7" and "interval $c-\nu \leqq x \leqq c+\nu$ of Fig.1-8" correspond by sight transfer, point $x=c$ of Fig.1-7 is microdomain point. In order to measure the rough estimate of length of microdomain radius, 2 sheets of papers are inserted parallel into the room sandwihed between road surface and the wheel from front side and back side. The distance of the 2 sheets of paper was about 11cm. The diameter of the wheel is about 54cm. As the car is front wheel drive, the length is about 3.7m, the width is about 1.6m, the height is about 1.5m. The weight of the car whithout load is about 1000kg, the load share of front and rear wheel is about 6:4, the load of one rear wheel is about 200kg. Compared to 54cm of diameter of wheel, microdomain radius ν is about 5.5cm. The length of about 5.5cm can be recognised by our eyes without any special equipment. Whether the length of microdomain radius about 5.5cm is deleted and it is considerd as a point, or it is not deleted and it is considered as a finite length, it depends on subjective views of observer. Sight transfer is a decision including not only the change of distance between the observer and observation object, but also the change of subjective view of the observer.

[Example]

Wheel load shown in Fig.1-7, Fig.1-8, is bailaterally symmetric with respect to vertical line AE which pass through the center E of the wheel, we also consider concentrated load which is not symmetric. The upper part of

Fig.1-9 is a concentrated load magnitude of P, which is applied through

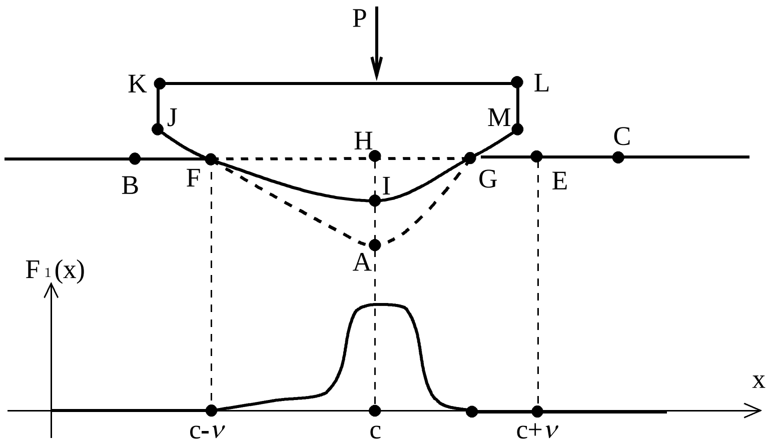

Fig.1-9 Concentrated load distributed not symmetrically

columb JFAGMLK with smooth base JFAGM. Base JFAGM deformed to JFIGM and road surface deformed to BFIGC, load is transmitted through curve FIG. Curve FIG is not bilaterally symmetric, in Fig.1-9, as FH is larger than HG, length FH be microdomain radius ν, choose point E which is HE$=\nu$, we consider straight line FE as microdomain. Rough sketch of transmitted distributed load $F_1(x)$ is not also symmetric shown in lower part of Fig.1-9. As for distributed load $F_1(x)$ of Fig.1-9, Formula1・20, Formula 1・21, Formula1・22 also hold. In Fig.1-8, as load is symmetric, point H divides straight line FG equally, in Fig.1-9, as load is not symmetric, point H does not divide straight line FG equally. The coordinate x$=$c of point H is inversly determined as numerical value c which satisfies Formula1・22. The coordinate of point A, I, H are the same x$=$c, point I is the point of application of concentrated load. If detail function $F_1(x)$ of Fig.1-9 which contain detail function $F_1(x)$ of Fig.1-8 satisfy Formula1・21, Formula1・22, they are equivlent each other, and they correspond outline function $f_1(x)$ of Formula1・18, Formula1・19. Length α, β of supports in Fig.1-4 are, in the meaning of microdomain radius of concentrated force, the same as length ν

in Fig.1-9.

(4) Concentrated moment

[Reaction moment]

Although it seems to be rare cace that concentrated moment acts on real structure as a load shown in Fig.1-1, reaction R_M of fixed support shown in Fig.1-10, is a concentrated moment which is often seen.

Fig.1-10 Reaction of fixed support

[Experimental loading]

Concentrated moment which acts on the structure as a load shown in Fig. 1-1 can be applied experimentally using equipment shown in upper part of Fig.1-11. Attached a lod of length e vertically at poiint A shown in the upper

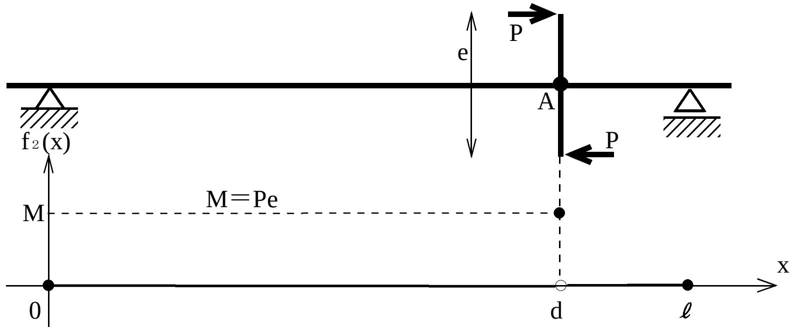

Fig.1-11 Concentrated moment by outline view

part of Fig.1-11, 2 forces are applied at upper end and lower end which magnitude are the same P and direction are opposite, then a concentrated moment of magnitude M$=$Pe is applied at point A. Set coordinate x along the member axis of the beam, coordinate of point A be x$=$d. If the function $f_2(x)$ expresses load, it is shown in lower part of Fig.1-11, and is expressed by Formula1・23, Formula1・24.

$$f_2(x)=0 \qquad (0 \leqq x < d、d < x \leqq \ell) \qquad 1・23$$

$f_2(x)=M \qquad (x=d) \qquad 1\cdot 24$

The unit of function $f_2(x)$ is the same Nm as concentrated moment M. Function $f_2(x)$ is outline function, and its domain of definition is interval $0\leqq x\leqq \ell$.

[Detail gaze]

When we approch seeing Fig.1-11 and gaze at near point A magnifying in detail, it becomes to look like the upper part of Fig.1-12. Using 2 equipments

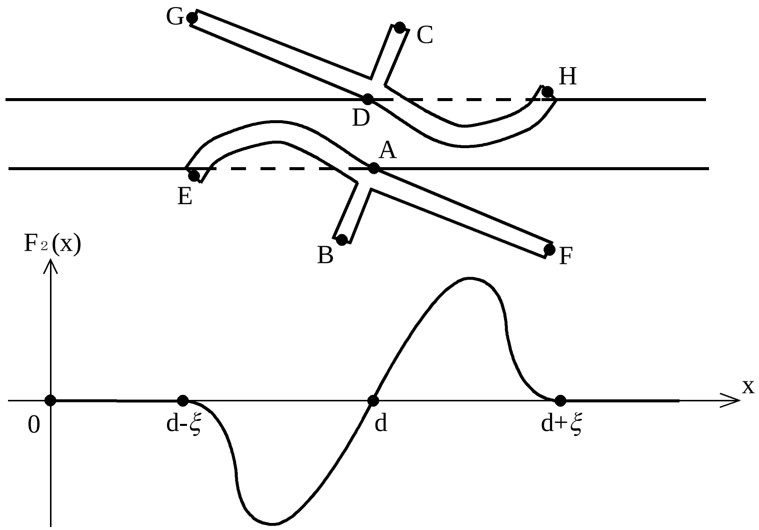

Fig.1-12 Conentrated moment by detail gaze

which are T shaped vertical rod BADC is attached to the beam. Horizontal plate EAF is pressed from underside, horizotal plate GDH is pressed from upside of the beam. When concentrated moment M is applied, horizontal plate EA deformed to the curve shown upper part of Fig.1-12, and the underside of the beam also deformed. In a similar way horizontal plate DH deformed to the curve and the topsurface of the beam also deformed. Load is tansmitted from loading equipment to the beam through the curve EA and the curve DH. Coordinates of point E and point G are $x=d-\xi$, and coordinates of point F and point H are $x=d+\xi$, load is distributed force in

the microdomain $d-\xi\leqq x\leqq d+\xi$, and the rough sketch is shown as function $F_2(x)$ of the lower part of Fig.1-12. The unit of function $F_2(x)$ is N/m, and is different from the unit of outline function $f_2(x)$. Function $F_2(x)$ is detailed function, and the domain of definition is interval $-\alpha\leqq x\leqq \ell+\beta$. As distributed force $F_2(x)$ is not distributed out of microdomain $d-\xi\leqq x\leqq d+\xi$, Formula1・25 holds.

$F_2(x)=0 \qquad (-\alpha\leqq x\leqq d-\xi、d+\xi\leqq x\leqq \ell+\beta) \qquad 1\cdot 25$

As moment of distributed force $F_2(x)$ about the point $x=d$ coincide with the value M of function $f_2(x)$ at the point $x=d$, Formula1・26 holds, and as total force of distributed force $F_2(x)$ is 0, Formula1・27 holds,

$$\int_{d-\xi}^{d+\xi} F_2(x)(x-d)dx=f_2(d)=M \qquad 1\cdot 26$$

$$\int_{d-\xi}^{d+\xi} F_2(x)dx=0 \qquad 1\cdot 27$$

[Equivalent detail function]

In the upper part of Fig.1-12, if the stiffnes of beam and loading equipment changes, curve EA and curve DH also change. If curve EA and curve GH differ, detail function $F_2(x)$ also differs, even if outline function $f_2(x)$ is the same. Correspondence between outline function $f_2(x)$ and detail function $F_2(x)$ is one to many. When different detail function $F_2(x)$ correspond to the same outline function $f_2(x)$, we would consider the different detail function $F_2(x)$ are equvalent each other.

[Outline view]

When we back away seeing Fig.1-12, 6 points E, A, F, G, D, H become to overlap and not to be distinguished gradually, it becomes look like point A of Fig.1-11. As "point $x=d$ of Fig.1-11" and "interval $d-\xi\leqq x\leqq d+\xi$ of Fig. 1-12" correspond by sight transfer, point $x=d$ of Fig.1-11 is microdomain point.

(5) Superposition of load

[Addition]

Although discussed separately distributed load, concentrated load, concentrated moment, as they are applied simultaneously in Fig.1-1, we consider superposition of load. Let function which is superposed outline function $f_0(x)$, $f_1(x)$, $f_2(x)$ be f(x), and let function which is superposed detail function $F_0(x)$, $F_1(x)$, $F_2(x)$ be F(x). As the 3 functions $F_0(x)$, $F_1(x)$, $F_2(x)$ have the same unit N/m, superposition can be expressed by simple adition, and can be written as Formula1･28.

$$F(x)=F_0(x)+F_1(x)+F_2(x) \qquad 1\cdot 28$$

But, as functions $f_0(x)$, $f_1(x)$, $f_2(x)$ do not have the same unit, superposition can not be expressed by simple addition. Considering that function f(x) has the same unit N/m as the unit of function F(x), add with unit, we may describe as Formula1･29, divide both sides by unit N/m, we obtain Formula 1･30.

$$f(x)N/m=f_0(x)N/m+f_1(x)N+f_2(x)Nm \qquad 1\cdot 29$$

$$f(x)=f_0(x)+f_1(x)m+f_2(x)m^2 \qquad 1\cdot 30$$

[3 components]

Unit m is the unit of coodinate x, and as coordinate x is the lateral axis of Fig.1-1, the upper and middle part of Fig.1-2, the lower part of Fig.1-3～Fig. 1-9, the lower part of Fig.1-11, 1-12, unit m is lateral axis unit. Unit m is unit of length, and cm and km and 尺 and foot are also unit of length. In order to generalize by avoiding to use specific unit, we replace m of Formula 1･30 with symbol and express as Formula1･31.

$$f(x)=f_0(x)+f_1(x)\,+f_2(x)\,{}^2 \qquad 1\cdot 31$$

The author propses that symbol is called lateral axis unit. The symbol is made by drawing orthogonal coordinate axis and marking a dot at the position indicating value 1 of lateral axis, becase the value 1 is unit. We can use capital letter "ヨ" or "y" or "H" as a symbol of lateral axis unit, capital letter "ヨ" is from "ヨコジク" in Japanese letter, capital letter "y" is from Japanese word "yokoziku" in Latain letter, capital letter "H" is from English word "Horizontal", we choose in this book. Formula1･31 seems to be vector expression, which basis vectors are unit 1, , ^{2} and components are function $f_0(x)$, $f_1(x)$, $f_2(x)$. As 1 means ^{0}, means ^{1}, exponent 0, 1, 2 of be order of component, the author proposes that fuction $f_0(x)$ is called 0-th order component, fuction $f_1(x)$ is called 1st order component, fuction $f_2(x)$ is called 2nd order component. If we express the same form as vector, it may be expressed by demarcated by comma , wrapped up by bracket { } like Formula1･32

$$f(x)=\{f_0(x),f_1(x),f_2(x)\} \qquad 1\cdot 32$$

[4 components]

By substituting Formula1-7 into Formula1･31, we obtain Formula1･33.

$$f(x)=f_h(x)+f_d(x)\,+f_1(x)\,+f_2(x)\,{}^2 \qquad 1\cdot 33$$

By substituting Formula1･6 into Formula1･32, we obtain Formula1･34.

$$f(x)=\{f_h(x),f_d(x),f_1(x),f_2(x)\} \qquad 1\cdot 34$$

Detail function F(x) has domain of definition of $-\alpha \leqq x \leqq \ell+\beta$ and is smooth function. Outline function f(x) is composed of 4 components, which are left contiuous component, step conponent, 1st component, 2nd component, and has domain of definition of $0 \leqq x \leqq \ell$.

[Similarity between complex number and function pseudo value]

The author proposes that expression using component as shown right side of Formula1･33 or Formula1･34 is called combind component expression of outline function. Component $f_h(x)$, $f_d(x)$, $f_1(x)$, $f_2(x)$ each is called individual component expression. Expression of Formula1･33 is called function pseudo value. As far as function f(x) of Formula1･35, function value is complex number expressed using real number unit 1 and imaginary number unit i.

$$f(x)=(x^2+4x+1)+(3x+2)i \qquad 1\cdot 35$$

Formula1･33 is also expressed using value 1 and step unit and lateral axis unit and square of lateral axis unit , and is similar to complex number. Compared to the fact that value of complex number has 2 unit, function pseudo value has 4 unit. Compared to the name of value of complex number, the name of function pseudo value is suitable. The expression of Formula1・34 is called function array.

(6) Summation of load

[Outline function]

Load produces an effect on a strcture to move. Fig.1-1 is slightly modified

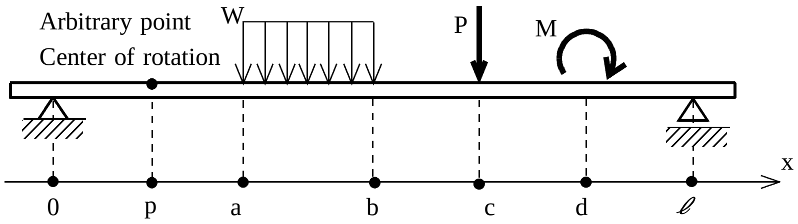

Fig.1-13 Summation of load within interval

and is put up again as Fig.1-13. An effect on the simple beam shown in Fig. 1-13 to move upward and downward is expressed by summation of load. Summation of load is shown as Formula1・36.

$$\int_{a+0}^{b} f_h(x)dx+f_1(c) \qquad 1\cdot 36$$

The 1st term of Formula1・36 is comprehensive integral of Formula1・14, and is total force W(b-a) of distributed load. The 2nd term of Formula1・36 is concentrated force P at point x＝c where concentrated force is applied. As far as Fig.1-13 is concerned, concentrated force is one within the interval $0 \leqq x \leqq \ell$, if several concentrated force are distributed, the sum of them is used. Concentrated moment is not used to calculate.

An effect on the simple beam shown in Fig.1-13 to rotate around arbitrary point x＝p is expressed by summation of moment of load. As the point x＝p can be set arbitrary, usually it is coincide to point x＝0 or x＝ℓ for the simple calculation. Summation of moment of load is shown as Formula1・37.

$$\int_{a+0}^{b} (x-p)f_h(x)dx+(c-p)f_1(c)+f_2(d) \qquad 1\cdot 37$$

The 1st term of Formula1・37 is comprehensive integral of moment of left continuous component $f_h(x)$ of distributed load. The 2nd term of Formula 1・37 is moment of concentrated force P at point x＝c where concentrated force is applied. The 3rd term of Formula1・37 is concentrated moment M at point x＝d where concentrated moment is applied. If several concentrated moment are distributed, the sum of them is used.

[Detail function]

As far as 3 kinds of load are concerned, summation calculation of load are different as shown in Formula1・36, summation calculation of moment of load are different as shown in Formula1・37. Although the way of calculation is superficially different, there must be common principle of summation calculation of load. When we use detail function, summation calculation of load is expressed by Formula1・38, summation calculation of moment of load is expressed by Formula1・39.

$$\int_{-\alpha}^{\ell+\beta} F(x)dx \qquad 1\cdot 38$$

$$\int_{-\alpha}^{\ell+\beta} (x-p)F(x)dx \qquad 1\cdot 39$$

As far as 3 kinds of load are concerned, Formula1・38, Formula1・39. is the same form. When Formula1・36 is deducted from Formula1・38, and Formula 1・37 is deducted from Formula1・39, Formula1・38and Formula1・39 are the common principle of summation of load.

[Deduct Formula1・36 from Formula1・38]

By substituting Formula1・28 into Formula1・38, we obtain Formula1・40.

$$\int_{-\alpha}^{\ell+\beta} F(x)dx=\int_{-\alpha}^{\ell+\beta} F_0(x)dx+\int_{-\alpha}^{\ell+\beta} F_1(x)dx+\int_{-\alpha}^{\ell+\beta} F_2(x)dx \qquad 1\cdot 40$$

By dividing integral interval of 1st term of right side of Formula1・40 into 3 parts, and into it substituting Formula1・15, Formula1・13, we obtain Formula

1・41.

$$\int_{-\alpha}^{\ell+\beta} F_0(x)dx = \int_{-\alpha}^{a-\mu} F_0(x)dx + \int_{a-\mu}^{b+\mu} F_0(x)dx + \int_{b+\mu}^{\ell+\beta} F_0(x)dx$$

$$= 0 + \int_{a-\mu}^{b+\mu} F_0(x)dx + 0$$

$$= \int_{a-0}^{b} f_h(x)dx \qquad 1・41$$

By dividing integral interval of 2nd term of right side of Formula1・40 into 3 parts, and into it substituting Formula1・20, Formula1・21, we obtain Formula 1・42.

$$\int_{-\alpha}^{\ell+\beta} F_1(x)dx = \int_{-\alpha}^{c-\nu} F_1(x)dx + \int_{c-\nu}^{c+\nu} F_1(x)dx + \int_{c+\nu}^{\ell+\beta} F_1(x)dx$$

$$= 0 + \int_{c-\nu}^{c+\nu} F_1(x)dx + 0 = f_1(c) \qquad 1・42$$

By dividing integral interval of 3rd term of right side of Formula1・40 into 3 parts, and into it substituting Formula1・25, Formula1・27, we obtain Formula 1・43.

$$\int_{-\alpha}^{\ell+\beta} F_2(x)dx = \int_{-\alpha}^{d-\xi} F_2(x)dx + \int_{d-\xi}^{d+\xi} F_2(x)dx + \int_{d+\xi}^{\ell+\beta} F_2(x)dx$$

$$= 0 + \int_{d-\xi}^{d+\xi} F_2(x)dx + 0 = 0 \qquad 1・43$$

By substituting Formula1・41, Formula1・42, Formula1・43 into Formula1・40, we obtain Formula1・44, then Formula1・36 is deducted from Formula1・38.

$$\int_{-\alpha}^{\ell+\beta} F(x)dx = \int_{a+0}^{b} f_h(x)dx + f_1(c) \qquad 1・44$$

[Deduct Formula1・37 from Formula1・39]

By substituting Formula1・28 into Formula1・39, we obtain Formula1・45.

$$\int_{-\alpha}^{\ell+\beta} (x-p)F(x)dx = \int_{-\alpha}^{\ell+\beta} (x-p)F_0(x)dx + \int_{-\alpha}^{\ell+\beta} (x-p)F_1(x)dx + \int_{-\alpha}^{\ell+\beta} (x-p)F_2(x)dx \qquad 1・45$$

By dividing integral interval of 1st term of right side of Formula1・45 into 3 parts, and into it substituting Formula1・15, we obtain Formula1・46.

$$\int_{-\alpha}^{\ell+\beta} (x-p)F_0(x)dx = \int_{-\alpha}^{a-\mu} (x-p)F_0(x)dx + \int_{a-\mu}^{b+\mu} (x-p)F_0(x)dx + \int_{b+\mu}^{\ell+\beta} (x-p)F_0(x)dx$$

$$= 0 + \int_{a-\mu}^{b+\mu} (x-p)F_0(x)dx + 0 \qquad 1・46$$

As function $(x-p)f_h(x)$ and function $f_h(x)$ have the same points $x=a$ and $x=b$ as microdomain points, Formula1・47 holds similar to Formula1・13.

$$\int_{a-\mu}^{b+\mu} (x-p)F_0(x)dx = \int_{a-0}^{b} (x-p)f_h(x)dx \qquad 1・47$$

Substituting Formula1・47 into Formula1・46, we obtain Formula1・48.

$$\int_{-\alpha}^{\ell+\beta} (x-p)F_0(x)dx = \int_{a-0}^{b} (x-p)f_h(x)dx \qquad 1・48$$

By dividing integral interval of 2nd term of right side of Formula1・45 into 3 parts, and into it substituting Formula1・20, Formula1・21, Formula1・22, we obtain Formula1・49.

$$\int_{-\alpha}^{\ell+\beta} (x-p)F_1(x)dx = \int_{-\alpha}^{c-\nu} (x-p)F_1(x)dx + \int_{c-\nu}^{c+\nu} (x-p)F_1(x)dx + \int_{c+\nu}^{\ell+\beta} (x-p)F_1(x)dx$$

$$= 0 + \int_{c-\nu}^{c+\nu} (x-p)F_1(x)dx + 0 = \int_{c-\nu}^{c+\nu} (x-c+c-p)F_1(x)dx$$

$$= \int_{c-\nu}^{c+\nu} (x-c)F_1(x)dx + (c-p)\int_{c-\nu}^{c+\nu} F_1(x)dx$$

$$= 0 + (c-p)f_1(c)$$

$$= (c-p)f_1(c) \qquad 1・49$$

By dividing integral interval of 3rd term of right side of Formula1・45 into 3 parts, and into it substituting Formula1・25, Formula1・26, Formula1・27, we obtain Formula1・50.

$$\int_{-\alpha}^{\ell+\beta} (x-p)F_2(x)dx = \int_{-\alpha}^{d-\xi} (x-p)F_2(x)dx + \int_{d-\xi}^{d+\xi} (x-p)F_2(x)dx + \int_{d+\xi}^{\ell+\beta} (x-p)F_2(x)dx$$

$$= 0 + \int_{d-\xi}^{d+\xi} (x-p)F_2(x)dx + 0$$

$$=\int_{d-\xi}^{d+\xi}(x-d+d-p)F_2(x)dx$$

$$=\int_{d-\xi}^{d+\xi}(x-d)F_2(x)dx+(d-p)\int_{d-\xi}^{d+\xi}F_2(x)dx$$

$$=f_2(d)+0$$

$$=f_2(d) \qquad 1\cdot50$$

By substituting Formula1・48, Formula1・49, Formula1・50 into Formula1・45, we obtain Formula1・51, then Formula1・37 is deducted from Formula1・39.

$$\int_{-\alpha}^{\ell+\beta}(x-d)F(x)dx=\int_{a+0}^{b}(x-d)f_h(x)dx+(c-p)f_1(c)+f_2(d) \qquad 1\cdot51$$

(7) Differential of sheering force and bennding moment

[Example]

Fig.1-14 shows a simple beam AB which is loaded a concentrated force.

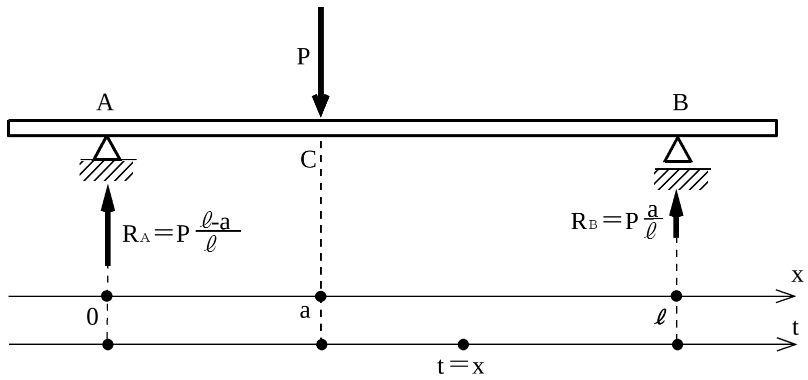

Fig.1-14 Simple beam loaded a concentrated load

Let the coordinate of point A be x＝0, point B be x＝ℓ, concentrated force P is applied at point C of coordinate x＝a. Reaction force at support R_A is expressed by Formula1・52, R_B is expressed by Formula1・53, and the units are the same N as concentrated force P.

$$R_A=P\frac{\ell-a}{\ell} \qquad 1\cdot52$$

$$R_B=P\frac{a}{\ell} \qquad 1\cdot53$$

[Outline function of external force of beam]

Outline function $g_1(x)$ of external force of beam which is superposed load and reaction of the beam in Fig.1-14 is expressed by Formula1・54 ～ Formula1・57.

$$g_1(x)=-P\frac{\ell\text{-}a}{\ell} \qquad (x=0) \qquad 1\cdot54$$

$$g_1(x)=0 \qquad (0<x<c、c<x<\ell) \qquad 1\cdot55$$

$$g_1(x)=P \qquad (x=c) \qquad 1\cdot56$$

$$g_1(x)=-P\frac{a}{\ell} \qquad (x=\ell) \qquad 1\cdot57$$

Domain of definition of function $g_1(x)$ is interval $0\leqq x\leqq\ell$. As for external force, downward direction is plus, so the parts of reaction R_A and reaction R_B of function $g_1(x)$ are minus. The unit of function $g_1(x)$ is N. As far as interval where concentrated force is not applied, we could consider that distributed force with magnitude of 0 is applied. Distributed force $g_z(x)$ is expressed by Formula1・58.

$$g_z(x)=0 \qquad (0<x<c、c<x<\ell) \qquad 1\cdot58$$

The unit of distributed force $g_z(x)$ is N/m, and is different from the unit N of the function $g_1(x)$.

[Detail function of external force of beam]

Detail function G(x) of external force which is superposed load and reaction force of the beam in Fig.1-14 is shown as Fig.1-15. When we soppose microdomain radius α of reaction force R_A and microdomain radius β of reaction force R_B, domain of definition of function G(x) is interval $-\alpha\leqq x\leqq\ell+\beta$. The unit of function G(x) is N/m, and is different from unit N of function $g_1(x)$, but is the same as the unit of distributed force $g_z(x)$ of Formula1・58.

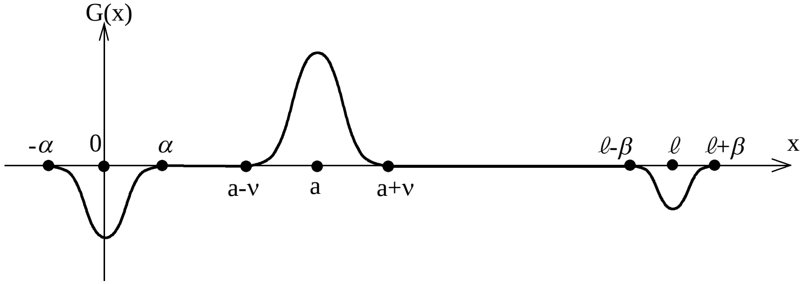

Fig.1-15　Detail function of external force of beam

[Sheering force]

Sheering force is defined that "the sheering force at an arbitrary point is what all external force which exist left side of the point are added such as upward derection is plus". As for external force, downward derection is plus, so it must be noted that the sign of external force and sheering force are opposite. Setting coordinate t same as coordinate x shown as Fig.1-14, summation of external force within the interval of left part of point t＝x on the axis t is calculated. Applied to detail function shown in Fig.1-15, sheering force S(x) is expressed by Formula1･59.

$$S(x) = -\int_{-\alpha}^{x} G(t)dt \qquad 1\cdot59$$

It is ambiguous that the interval of left of point t＝x is interval $-\alpha \leqq t \leqq x$ or

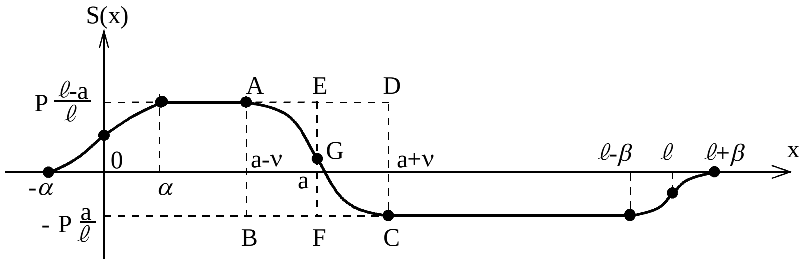

Fig.1-16　Detail function of sheering force

interval $-\alpha \leqq t < x$, but whichever interval is used, it is expressed by integral shown as Formula1･59. The unit of the function S(x) is N. Sheering force

S(x) is illustrated as Fig.1-16. Function S(x) of formula1･59 is uniquely defined at all of the points of interval $-\alpha \leqq x \leqq \ell+\beta$. When we back away seeing Fig.1-16, line AB and line DC become to overlap and not to be distinguished gradually, rectangular ABCD and line EF become not to be distinguished, interval $-\alpha \leqq x \leqq \alpha$ and interval $\ell-\beta \leqq x \leqq \ell+\beta$ become to be seen like vertical line. Then left continuous component $s_h(x)$ and step component $s_d(x)$ of outline function are expressed by Formula1･60 ～ Formula1･65.

$$s_h(x) = 0 \qquad (x = 0) \qquad 1\cdot60$$

$$s_h(x) = P\frac{\ell-a}{\ell} \qquad (0 < x \leqq a) \qquad 1\cdot61$$

$$s_h(x) = -P\frac{a}{\ell} \qquad (a < x \leqq \ell) \qquad 1\cdot62$$

$$s_d(x) = P\frac{\ell-a}{\ell} \qquad (x = 0) \qquad 1\cdot63$$

$$s_d(x) = -P \qquad (x = a) \qquad 1\cdot64$$

$$s_d(x) = P\frac{a}{\ell} \qquad (x = \ell) \qquad 1\cdot65$$

Unit of function $s_h(x)$ and function $s_d(x)$ are the same N as unit of function S(x). Domain of definition of outline function of Formula1･60 ～ Formula 1･65 is interval $0 \leqq x \leqq \ell$.

[Hitherto existing explanation]

As hitherto existing textbook[1)] calculates summation of external forces of the beam within the interval of the left of point t＝x on the axis t of Fig. 1-14, the fact that it is ambiguous interval $-\alpha \leqq t \leqq x$ or interval $-\alpha \leqq t < x$ is betrayed, the result of calculation at the application point of concentrated force are different, sheering force $s_z(x)$ is expressed by Formula1･66, Formula1･67.

$$s_z(x) = P\frac{\ell-a}{\ell} \qquad (0 < x < a) \qquad 1\cdot66$$

$$s_z(x) = -P\frac{a}{\ell} \qquad (a < x < \ell) \qquad 1\cdot67$$

The values of sheering force $s_z(x)$ at application points $x=0$, $x=a$, $x=\ell$ of concentrated force can not be determined uniquely, Formula1･66, Formula 1･67 avoid to explain about $s_z(0)$, $s_z(a)$, $s_z(\ell)$. Outline function of Formula 1･60 ～ Formula1･65 explains about whole interval of domain of definition $0 \leqq x \leqq \ell$ which contains points $x=0$, $x=a$, $x=\ell$. Function $s_h(x)$ of Formula 1･60～Formula1･62 and function $s_z(x)$ of Formula1･66, Formula1･67 are slightly different.

[Bending moment]

Bending moment is defined that "the bending moment at an arbitrary point is what all moment of external forces which exist left side of the point about the point are added such as clockwise rotation is plus". Summation of moments about the point $t=x$ of external forces which exist left side of point $t=x$ on the axis t of Fig.1-14 is calculated. Applying the detail function of Fig.1-15, bending moment M(x) is expressed by Formula1･68.

$$M(x)=\int_{-\alpha}^{x}(t-x)G(t)dt \qquad 1･68$$

Although it is ambiguous that the interval of left of point $t=x$ is interval $-\alpha \leqq t \leqq x$ or interval $-\alpha \leqq t < x$, as detail function G(x) is continuous, whichever interval is used, it is expressed by integral shown as Formula1･68. Bending moment M(x) is is illustrated as Fig.1-17. Function M(x) of

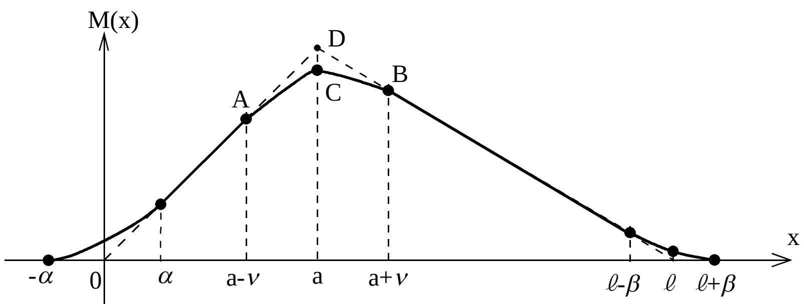

Fig.1-17 Detail function of bending moment

Formula1･68 is uniquely defined at all of the points of interval $-\alpha \leqq x \leqq \ell+\beta$.

When we back away seeing Fig.1-17, 3 points A, C, B become to overlap and not to be distinguished from point D gradually, and interval $-\alpha \leqq x \leqq \alpha$ and interval $\ell-\beta \leqq x \leqq \ell+\beta$ also look like points. Then left continuous component $m_h(x)$ and step component $m_d(x)$ of outline function are expressed by Formula1･69 ～ Formula1･71. The unit of function $m_h(x)$ and function $m_d(x)$ are Nm.

$$m_h(x)=P\frac{\ell-a}{\ell}x \qquad (0 \leqq x \leqq a) \qquad 1･69$$

$$m_h(x)=-P\frac{a}{\ell}(x-\ell) \qquad (a \leqq x \leqq \ell) \qquad 1･70$$

$$m_d(x)=0 \qquad (0 \leqq x \leqq \ell) \qquad 1･71$$

Domain of definition of outline function of Formula1･69 ～ Formula1･71 is interval $0 \leqq x \leqq \ell$.

[Hitherto existing explanation]

As hitherto existing textbook[1)] calculates summation of moments of external forces of the beam within the interval of the left of point $t=x$ on the axis t of Fig.1-14, although it is ambiguous that interval summation is whether interval $-\alpha \leqq t \leqq x$ or interval $-a \leqq t < x$, the result of calculation at the application point of concentrated force, are the same, bending moment $m_z(x)$ is expressed by Formula1･72, Formula1･73.

$$m_z(x)=P\frac{\ell-a}{\ell}x \qquad (0 \leqq x \leqq a) \qquad 1･72$$

$$m_z(x)=-P\frac{a}{\ell}(x-\ell) \qquad (a \leqq x \leqq \ell) \qquad 1･73$$

Bending moment $m_z(a)$ at point $x=a$ are calculated by Formula1･72, Formula1･73, as the calculation results of both Formula are the same, Formula1･72, Formula1･73 is actually the same with Formula1･69, Formula 1･70.

[Differential at application point of concentrated force]

As far as distributed load $g_z(x)$ expressed by Formula1･58 and sheering force $s_z(x)$ expressed by Formula1･66, Formula1･67 and bending moment

$m_z(x)$ expressed by Formula1･72, Formula1･73, the relation of Formula 1･74, Formula1･75 is known.

$$\frac{d}{dx}s_z(x)=-g_z(x) \qquad 1･74$$

$$\frac{d}{dx}m_z(x)=s_z(x) \qquad 1･75$$

But, it is indifferentiable at application points x＝0, x＝a, x＝ℓ of concentrated force, so that Formula1･74, Formula1･75 do not hold. Sheering force S(x) expressed by Formula1･59 and bending moment M(x) expressed by Formula1･68 are differentiable even in the microdomain $-\alpha\leqq x\leqq\alpha$, $a-\nu\leqq x\leqq a+\nu$, $\ell-\beta\leqq x\leqq\ell+\beta$ corresponding points x＝0, x＝a, x＝ℓ, it is calculated as Formula1･76, Formula1･77.

$$\frac{d}{dx}S(x)=-G(x) \qquad 1･76$$

$$\frac{d}{dx}M(x)=\frac{d}{dx}\int_{-\alpha}^{x}(t-x)G(t)dt=\frac{d}{dx}\int_{-\alpha}^{x}tG(t)dt-\frac{d}{dx}x\int_{-\alpha}^{x}G(t)dt$$

$$=xG(x)-(\frac{d}{dx}x)\int_{-\alpha}^{x}G(t)dt-x\frac{d}{dx}\int_{-\alpha}^{x}G(t)dt$$

$$=xG(x)-\int_{-\alpha}^{x}G(t)dt-xG(x)=-\int_{-\alpha}^{x}G(t)dt=S(x) \qquad 1･77$$

Sheering force and bending moment in the hitherto existing structural mechanics are indifferentiable at the application points of concentrated force. Sheering force and bending moment expressed by detail function are, within the microdomain corresponding application point of concentrated force, smooth as the curve AGC of Fig.1-16 and the curve ACB of Fig.1-17, and differentiable.

Chapter 2 Functional Type Hyper Function

(1) Comprehensive integral within an interval which contains a point inintegrable

[Definition]

Even if function f(x) is inintegrable at point x＝c, when the limit of the 1st hand side of Formua2･1 converges using parameters ε、ζ, function f(x) is said to be comprehensively integrable within the interval a≦x≦b. Provided it is assumed to be a＜c＜b.

$$\lim_{\varepsilon\to 0}\int_{a}^{c-\varepsilon}f(x)dx+\lim_{\zeta\to 0}\int_{c+\zeta}^{b}f(x)dx=\int_{a}^{c-0}f(x)dx+\int_{c+0}^{b}f(x)dx$$

$$=\int_{a}^{c}f(x)dx+\int_{c}^{b}f(x)dx=\int_{a}^{b}f(x)dx \qquad 2･1$$

The point x＝c of Formula2･1 is inintegrable but comprehensively integrable point. Points which are inintegrable but comprehensively integrable exist discretely in the integrable points which are continuously existing. In order to show clearly that the 2nd hand side of Formua2･1 is comprehensive integral, it expresses the end points of integral with c-0 or c+0. But when it is not necessary to show clearly, it is expressed like the 3rd hand side or 4th hand side of Formula2･1. In the expression of 4th hand side, inintegrable poit x＝c is not recognized. Function $f_h(x)$ of Formula1･1, Formula1･2 is right inintegrable at points x＝a, x＝b, but comprehensively integrable.

$$f_h(x)=0 \qquad (0\leqq x\leqq a、b<x\leqq\ell) \qquad 1･1(again)$$

$$f_h(x)=W \qquad (a<x\leqq b) \qquad 1･2(again)$$

As for integral interval a≦x≦b, it is inintegrable at point x＝a, so comprehensive integrable of Formula1･14(reffer to page 6) is used.

[Example]

As for the calculation of 1st hand side of Formula2･1, interval $c-\varepsilon\leqq x\leqq c+\zeta$ is excluded from integral calculation. As considered limit